

Ex4) A co the row pre

DEFINITION: A Series is the sum of the terms of a sequence.

Sigma Notation or (Summation Notation) This symbol means "Add" $\rightarrow \sum$ It's called "Sigma"

$\sum_{n=1}^k a_n$ The variable below sigma (n in this case) is called the "index"
 The number below sigma (1 in this case) is which term begins the series, called the "lower bound"
 The number above sigma (k in this case) is which term ends the series, called the "upper bound"
 The expression to the right of sigma (a_n in this case) is the **explicit formula** used to generate the terms of the series.

Ex 1) Find the sum of each finite sequence.

- (a) $\sum_{n=1}^5 n^2$ $1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$
- (b) $\sum_{n=3}^5 \frac{1}{n}$ $\frac{47}{60}$
- (c) $\sum_{n=5}^{10} n$ 45
- (d) $\sum_{n=1}^6 2$ 12 $2+2+2+2+2+2$

Ex2) Write the following sums using sigma notation:

- a) $1+4+9+16+25+36+49+64+81$ $\sum_{x=1}^9 (x^2)$
- b) $2+4+6+8+10+12$ $\sum_{n=1}^6 2+2(n-1)$
- c) $625+125+25+\dots$ $\sum_{n=1}^{\infty} 625(\frac{1}{5})^{n-1}$
- d) $6+2-2-6-10-14-18-22$ $\sum_{n=1}^8 6-4(n-1)$
- e) $729+243+81+27+9+3$ $\sum_{n=1}^6 729(\frac{1}{3})^{n-1}$
- f) $8+27+64+125$ $\sum_{x=2}^5 x^3$

FINITE ARITHMETIC SERIES $S_n = \sum_{k=1}^n a_k = \frac{n}{2}(a_1 + a_n)$

Ex3) Find each sum. $\sum_{n=1}^7 3+4(n-1)$
 $a_1 + d(n-1)$

a) $3+7+11+15+19+23+27$

$S_n = \frac{7}{2}(3+27)$
 $S_7 = 105$

b) $1+3+5+7+\dots+49$

$S_n = \frac{25}{2}(1+49)$
 $= 625$

$49 = 1+2(n-1)$
 $48 = 2(n-1)$
 $24 = n-1$
 $n = 25$

Ex4) A corner section of a stadium has 8 seats along the front row. Each successive row has 2 more seats than the row preceding it. If the top row has 24 seats how many seats are in the entire section?

$$24 = 8 + 2(n-1)$$

$$n = 9$$

$$\sum_{n=1}^9 8 + 2(n-1)$$

$$S_n = \frac{9}{2}(8+24)$$

$$= 144$$

Sum of a Finite Geometric Series:

$$S_n = a_1 \frac{1-r^n}{1-r}$$

Sum of an Infinite Geometric Series:

$$\sum_{k=1}^{\infty} a_1 \cdot r^{n-1} = \frac{a_1}{1-r}, \text{ if \& only if } |r| < 1$$

$$S = \frac{a_1}{1-r}$$

If an infinite series has a sum it is Convergent.

If it does not, it is Divergent.

Ex5) Find the sum of each geometric series.

a) $\sum_{n=1}^9 4 \left(-\frac{1}{3}\right)^{n-1}$

$$S_9 = 4 \left(\frac{1 - (-\frac{1}{3})^9}{1 - (-\frac{1}{3})} \right) = 3$$

b) $5 + 15 + 45 + \dots + 98415$

$$\frac{98415}{5} = \frac{5(3)^{x-1}}{5}$$

$$19683 = 3^{x-1}$$

$$\log_3 19683 = x-1$$

$$x = 10$$

$$\sum_{x=1}^{10} 5(3)^{x-1} = 147620$$

Ex6) Determine if each of the following series converges. If the series converges, find the sum.

a) $\sum_{n=1}^{\infty} 3(0.75)^{n-1}$ C

$$S = \frac{3}{1-0.75} = \frac{3}{0.25} = 12$$

b) $\sum_{n=0}^{\infty} \left(-\frac{4}{5}\right)^{n-1}$ C

or $\frac{a_1}{1-r} = \frac{1}{1 - (-\frac{4}{5})} = \frac{5}{9} + \frac{-5}{4} = \frac{-25}{36}$

or $\frac{-\frac{5}{4}}{1 - (-\frac{4}{5})} = \frac{-25}{36}$

c) $\sum_{n=1}^{\infty} \left(\frac{\pi}{2}\right)^n$ Divergent

d) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1}$ C

$$\frac{1}{1-\frac{1}{2}} = 2$$

Unit 8 Pg. 7 HW

(1) $\sum_{x=1}^5 3x = 45$ (b) $\sum_{x=1}^{80} 2-x = -3080$ (c) $\sum_{x=3}^{24} (1-2x) = -572$ (d) $\sum_{x=1}^5 7 = 35$

(2) $38 + 34 + 30 + \dots + 2$
 $a_2 = 38 - 4(n-1)$
 $-36 = -4(n-1)$
 $n = 10$
 $\sum_{n=1}^{10} 38 - 4(n-1) = 200$

(3) $2 + 5 + 8 + 11$
 $\sum_{n=1}^{60} 2 + 3(n-1) = 5430$

(4) (a) $a_{72} = 20 + 4(72-1)$
 $a_{72} = 304$

(b) $\sum_{n=1}^{85} 20 + 4(n-1) = 15980$

(c) $\sum_{n=32}^{64} 20 + 4(n-1) = 6864$

(5) $1 = 15 - 2(n-1)$
 $n = 8$
 $S_8 = \frac{8}{2}(15+1)$
 $S_8 = 64$

(6) (a) $\sum_{x=1}^8 -2(-2)^{x-1} = 170$

(7) $S = \frac{256}{1 - \frac{1}{4}} = \frac{1024}{3} = 204.8$

(b) $\sum_{x=1}^6 -1(2)^{x-1} = -63$

(8) $\sum_{x=1}^8 (-6)^{x-1} = -239945$

(9) (a) Conver
 $\frac{2}{1 - \frac{2}{3}} = 6$

(b) Conver
 $\frac{8}{1 - \frac{1}{2}} = 16$

(c) Conver
 $\frac{2}{1 - \frac{1}{4}} = \frac{8}{3}$

(10) $\sum_{x=1}^{64} (1(2)^{x-1}) = 1.8 \times 10^{19}$

(d) Divergent

(e) Conver
 $\frac{6}{1 - \frac{1}{3}} = 9$

(f) Divergent

(11) (A) $a_5 = 20000(1.06)^4 = 26784.54$
 $a_5 = 22000(1.03)^4 = 24761.19$

(1)

(B) $S_5 : \sum_{x=1}^5 20000(1.06)^{x-1} = 112741.86$

$S_5 : \sum_{x=1}^5 22000(1.03)^{x-1} = 116800.99$

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