Precalculus Unit 8 Notes—Sequences

Definition: A Sequence is an ordered progression of numbers. This progression can be finite (meaning it ends), for example {3, 6, 9, 12, …, 21}. Or, it can be infinite, for example {3, 6, 9, 12, ….}.

Notation: a_n is used to denote a term in a sequence. The a alone actually has no value, however the n has a very significant meaning. It indicates the number of the term in the sequence being referred to.

There are 2 ways to define these sequences

explicit & recursive

The **explicit definition** is like a formula.

Ex1) Find the first four terms of the given sequence.

a) $\boxed{a_n = 2n+3}$

5, 7, 9, 11
a_1 a_2 a_3 a_4
n=1 n=2

b) $a_n = 3 \cdot 2^n$

6, 12, 24, 48
a_1 a_2 a_3 a_4

c) $a_n = n + \frac{1}{n}$

2, $2\frac{1}{2}$, $3\frac{1}{3}$, $4\frac{1}{4}$
a_1 a_2 a_3 a_4

d) $a_n = n^3 + 1$

2, 9, 28, 65
a_1 a_2 a_3 a_4

e) $a_n = 3 - 7n$

-4, -11, -18, -25
a_1 a_2 a_3 a_4

f) $a_n = (-2)^n$

-2, 4, -8, 16
a_1 a_2 a_3 a_4

The **recursive definition** has 2 parts:

(1) a term with which to begin
(2) a symbolic description of how the successive terms are related.

Ex2) Find the indicated terms of the given sequence.

a) $a_1 = 6$, $a_n = 4 + a_{n-1}$
$4 + a_{2-1} \rightarrow 4 + a_1$

10, 14, 18, 22
a_2 a_3 a_4 a_5

b) $a_1 = 9$, $a_n = \frac{1}{3} \cdot a_{n-1}$

3, 1, $\frac{1}{3}$, $\frac{1}{9}$
a_2 a_3 a_4 a_5

c) $a_1 = 1, a_2 = 2$, $a_n = a_{n-1} + a_{n-2}$

2, 3, 5, 8
a_2 a_3 a_4 a_5

d) $a_1 = 4$, $a_n = 5 \cdot a_{n-1} + 2$

22, 112, 562, 2812
a_2 a_3 a_4 a_5

e) $a_1 = 1$, $a_n = \left(-\frac{1}{3}\right)^n \cdot a_{n-1}$

$\frac{1}{9}$, $\frac{-1}{3^5}$, $\frac{1}{3^9}$, $\frac{-1}{3^{14}}$
a_2 a_3 a_4 a_5

f) $a_1 = 1, a_2 = 2$, $a_n = a_{n-1} \cdot a_{n-2}$

2, 2, 4, 8
a_2 a_3 a_4 a_5

Although it is possible to work with many different types of sequences, there are 2 that are most common.

arithmetic (where there is a common difference between each term) and geometric (where there is a common ratio between each pair of terms).

$a_n = a_1 \cdot r^{n-1}$

Explicit **ARITHMETIC:**

$\boxed{a_n} = a_1 + d(n-1)$ **, where d is the difference between each term** (called the common difference)

Ex3) State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the nth term of the sequence in terms of n.

a) 17, 21, 25, 29, …   +4 +4 +4

type: Arith.

$a_n = 17 + 4(n-1)$

b) 8, 12, 18, 27, … $\frac{12}{8} = \frac{3}{2}$ $\frac{18}{12} = \frac{3}{2}$ $\frac{27}{18} = \frac{3}{2}$

type: Geometric

$a_n = 8 \cdot \left(\frac{3}{2}\right)^{n-1}$

c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

type: Neither

$a_n = \frac{n}{n+1}$

d) 11, 101, 1001, 10001, …

type: Neither

$a_n = 10^n + 1$

e) 100, –50, 25, –12.5,…

type: Geometric

$a_n = 100 \cdot \left(\frac{-1}{2}\right)^{n-1}$

f) 1, 4, 9, 16, …

type: Neither

$a_n = n^2$

g) $\frac{2}{1}, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \ldots$

type: Neither

$a_n = \frac{n+1}{n^2}$

h) 2a – 2b, 3a – b, 4a, 5a + b, …

type: Arithmetic

$a_n = 2a - 2b + (a+b)(n-1)$

Ex4) State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the nth term of the sequence in terms of n.

a) $a_1 = 8$, $a_n = \frac{1}{2} \cdot a_{n-1}$ 8, 4, 2, 1

type: Geometric

$a_n = 8 \cdot \left(\frac{1}{2}\right)^{n-1}$

b) $a_1 = 6$, $a_n = a_{n-1} + 10$ 6, 16, 26

type: Arithmetic

$a_n = 6 + 10(n-1)$

c) $a_1 = \frac{1}{2}$, $a_n = \frac{n}{n+1}(a_{n-1} + 1)$ $\frac{1}{2}$, $\frac{2}{3} \cdot \frac{3}{2} = 1$, $\frac{3}{4}(1+1) = \frac{3}{2}$, $\frac{4}{5}\left(\frac{3}{2}+1\right) = 2$

type: Arithmetic

$a_n = \frac{1}{2} + \frac{1}{2}(n-1)$

d) $a_1 = 1$, $a_n = a_{n-1} + 2n - 1$ 1, 1+3=4, 4+5=9

type: Neither

$a_n = n^2$

e) $a_1 = 3$, $a_n = -2 \cdot a_{n-1}$ 3, -6, 12

type: Geometric

$a_n = 3 \cdot (-2)^{n-1}$

f) $2^{\frac{2}{3}}, 2^{\frac{5}{3}}, 2^{\frac{8}{3}}, \ldots$

type: Geometric

$a_n = 2^{\frac{2}{3}} \cdot 2^{n-1}$

Ex5) Find the indicated term of each arithmetic sequence:

a) $a_1 = 15$, $a_2 = 21$, $a_{20} = ?$

$d = 6$

$a_{20} = 15 + 6(20-1)$

$a_{20} = 129$

b) $a_1 = 15$, $a_2 = 7$, $a_{20} = ?$

$d = -8$

$a_{20} = 15 - 8(20-1)$

$a_{20} = -137$

Ex6) How many terms are in the finite arithmetic sequence

a) 18, 24, …, 336

(1) $336 = 18 + 6(n-1)$

(2) $\frac{318}{6} = \frac{6(n-1)}{6}$

(3) $53 = n - 1$

(4) $\boxed{n = 54}$

b) 178, 170, …, 2

$2 = 178 - 8(n-1)$

$\frac{-176}{-8} = \frac{-8(n-1)}{-8}$

$22 = n - 1$

$\boxed{n = 23}$

Ex7) Find the number of multiples of 7 between 30 and 300.

$294 = 35 + 7(n-1)$

$n = 38$

Geometric Sequence:

$a_n = a_1(r)^{n-1}$, where a_n is the nth term, r is the common ratio, & a_1 is the 1st term.

Ex8) Write an explicit representation of the pattern. Then find the 15th term.

$\frac{1}{243}, \frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \ldots$

$r = 3$

$a_{15} = \frac{1}{3^5} \cdot 3^{15-1}$

$a_{15} = 3^9$

$a_n = \frac{1}{243} \cdot 3^{n-1}$

Ex9) Given that $a_2 = 3$ & $a_5 = 24$ write an explicit formula if the sequence is a) arithmetic & b) geometric. Then find the values of a_3, and a_4 in each situation.

a) $d = \frac{24-3}{5-2} = 7$

$\frac{-4}{1}$ $\frac{3}{2}$ $\frac{10}{3}$ $\frac{17}{4}$ $\frac{24}{5}$

$a_n = -4 + 7(n-1)$

$a_3 = 10$ $a_4 = 17$

(These are called the Arithmetic means between a_2 & a_5)

b) $r = \sqrt[5-2]{\frac{24}{3}} = 2$

$\frac{3/2}{a_1}$ $\frac{3}{a_2}$ $\frac{6}{a_3}$ $\frac{12}{a_4}$ $\frac{24}{a_5}$

$a_n = \frac{3}{2} \cdot 2^{n-1}$

$a_3 = 6$ $a_4 = 12$

(These are called the Geometric means between a_2 & a_5)

Unit 8 Pg. 4 HW

① 35, 32, 29, 26
Arith.

② Neither
1, 8, 27, 64...

③ -1, 6, -36, 216...
Geometric

② $a_n = -11 + 7n$
$a_1 = -11 + 7(1) = -4$
$a_2 = -11 + 7(2) = 3$
$a_3 = -11 + 7(3) = 10$
$a_4 = -11 + 7(4) = 17$
$a_5 = -11 + 7(5) = 24$
$a_{34} = -11 + 7(34) = 227$

③ $a_1 = 28$ $d = 10$
$a_n = 28 + 10(n-1)$
$a_1 = 28 + 10(1-1) = 28$
$a_2 = 28 + 10(2-1) = 38$
$a_3 = 28 + 10(3-1) = 48$
$a_4 = 28 + 10(4-1) = 58$
$a_5 = 28 + 10(5-1) = 68$

④ $a_{38} = -53.2$ $d = -1.1$
$-53.2 = a_1 - 1.1(38-1)$
$a_1 = -12.5$
$a_n = -12.5 - 1.1(n-1)$
$a_1 = -12.5$
$a_2 = -13.6$
$a_3 = -14.7$
$a_4 = -15.8$
$a_5 = -16.9$

⑤ $a_1 = \frac{3}{5}$ $d = -\frac{1}{3}$
$a_n = a_{n-1} - \frac{1}{3}$
$a_1 = \frac{3}{5}$
$a_2 = \frac{4}{15}$
$a_3 = -\frac{1}{15}$
$a_4 = -\frac{2}{5}$

⑥ $a_{21} = -1.4$ $d = .6$
$a_n = a_{n-1} + .6$
$a_{21} = -1.4$
$a_{22} = -.8$
$a_{23} = -.2$
$a_{24} = .4$

⑦ $a_{18} = 3362$ $a_{38} = 7362$
$d = \frac{7362 - 3362}{38 - 18} = \frac{4000}{20} = 200$
$a_n = a_{n-1} + 200$

⑧ $a_n = 3^{n-1}$
$a_1 = 1$
$a_2 = 3$
$a_3 = 3^2$
$a_4 = 3^3$
$a_5 = 3^4$
$a_8 = 3^7$

⑨ $a_n = a_{n-1} \cdot 2$
$a_1 = 2$
$r = 2$
$a_n = 2(2)^{n-1}$
$a_1 = 2$ $a_4 = 16$
$a_2 = 4$ $a_5 = 32$
$a_3 = 8$

⑩ $a_1 = .8$ $r = -5$
$a_n = .8(-5)^{n-1}$
$a_1 = .8$
$a_2 = -4$
$a_3 = 20$
$a_4 = -100$
$a_5 = 500$

Unit 8 Pg. 4 HW

(11) $a_1 = -4 \quad r = 6$

$a_n = a_{n-1} \cdot 6$

$a_1 = -4$

$a_2 = -24$

$a_3 = -144$

$a_4 = -864$

(12) $a_4 = 25 \quad r = -5$

$a_n = a_{n-1} \cdot (-5)$

$a_n = \frac{-24}{125} (-5)^{n-1}$

$24 = a_1 (-5)^{4-1}$

$a_1 = \frac{-24}{125}$

$a_1 = \frac{-24}{125}$

$a_2 = \frac{+24}{25}$

$a_3 = \frac{-24}{5}$

$a_4 = 24$

$a_5 = -120$

(13) $a_4 = -12 \quad a_5 = -6$

$r = \frac{1}{2}$

$a_n = a_{n-1} \cdot \left(\frac{1}{2}\right)$

$a_4 = -12$

$a_5 = -6$

$a_6 = -3$

$a_7 = -\frac{3}{2}$

$\boxed{a_8 = -\frac{3}{4}}$

(12) $a_4 = 25 \quad r = -5$

$a_n = a_{n-1} \cdot (-5)$

$a_n = -\frac{1}{5} (-5)^{n-1}$

$25 = a_1 \cdot (-5)^{4-1}$

$\frac{25}{-125} = \frac{a_1 \cdot (-5)^{4-1}}{-125}$

$-\frac{1}{5} = a_1$

$a_1 = -\frac{1}{5}$

$a_2 = 1$

$a_3 = -5$

$a_4 = 25$

$a_5 = -125$