Precalculus Unit 7 *Notes—Polar Coordinates*

A **polar coordinate system** is a plane with a point O, the **pole**, and a ray from O, the **polar axis**. Each point P in the plane is assigned as **polar coordinates** as follows: r is the **directed distance** from O to P and θ is the **directed angle** whose initial side is on the polar axis and whose terminal side is on the line OP.

As in trigonometry, we measure θ as positive when moving counterclockwise and negative when moving clockwise. If $r > 0$, then P is on the terminal side of θ. If $r < 0$, then P is on the terminal side of $\theta + \pi$. We can use radian or degree measure for the angle θ.

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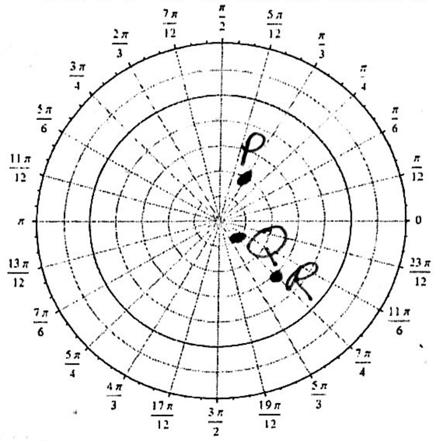
EXAMPLE 1 Plotting Points in the Polar Coordinate System

Plot the points with the given polar coordinates.

(a) $P(2, \pi/3)$

(b) $Q(-1, 3\pi/4)$

(c) $R(3, -45°)$



<u>NOTE</u>: Each polar coordinate pair determines a unique point. However, the polar coordinates of a point P in the plane are not unique.

Coordinate Conversion Equations

Let the point P have polar coordinates (r, θ) and rectangular coordinates (x, y). Then

$$x = r\cos\theta, \qquad r^2 = x^2 + y^2,$$
$$y = r\sin\theta, \qquad \tan\theta = \frac{y}{x}.$$

EXAMPLE 2 Converting from Polar to Rectangular Coordinates

Find the rectangular coordinates of the points with the given polar coordinates.

(a) $P(3, 5\pi/6)$

$\left(3\cos\frac{5\pi}{6},\ 3\sin\frac{5\pi}{6}\right)$

$\left(-\frac{3\sqrt{3}}{2},\ \frac{3}{2}\right)$

(b) $Q(2, -200°)$

$(2\cos(-200),\ 2\sin(-200))$

$(-1.9,\ .7)$

EXAMPLE 3 Converting from Rectangular to Polar Coordinates

Find two polar coordinate pairs for the points with given rectangular coordinates.

(a) $P(-1, 1)$ $x^2 + y^2 = r^2$

$r = \sqrt{2}$

$\tan\theta = \frac{1}{-1}$

$(\sqrt{2}, -225°)$ $\theta = -45$ convert to correct quadrant $\rightarrow 135°$

$(-\sqrt{2}, 315°)$

$(\sqrt{2}, 135°)$ $(-\sqrt{2}, -45°)$

(b) $Q(-3, 0)$ $r^2 = x^2 + y^2$

$r = 3$

$\tan\theta = \frac{0}{-3}$

$\theta = 0°$ convert to correct quadrant $\rightarrow 180°$

$(3, 180°)$ $(3, -180°)$

$(-3, 0°)$ $(-3, 360°)$

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Equation Conversion

We can use the Coordinate Conversion Equations to convert polar form to rectangular form and vice versa. Just as with parametric equations, the domain of a polar equation in *r* and θ is understood to be all values of θ for which the corresponding values of *r* are real numbers. You must also select a value for θmin and θmax to graph in polar mode.

Converting from Polar Form to Rectangular Form

EXAMPLE 4 Convert each polar equation to rectangular form.

A. $r = 4\sec\theta$

$r = \frac{4}{\cos\theta}$

$4 = r\cos\theta$

$x = 4$

B. $r(r = 4\cos\theta)$

$r^2 = 4r\cos\theta$

$x^2 + y^2 = 4x$

$x^2 - 4x + 4 + y^2 = 0 + 4$

$(x-2)^2 + y^2 = 4$

Circle $(2, 0)$ $r = 2$

C. $r = 3\cos\theta$

D. $r^2 = -3\sec\theta$

$\frac{1}{r} \cdot r^2 = \frac{-3}{\cos\theta} \cdot \frac{1}{r}$

$r = \frac{-3}{r\cos\theta}$

$(r)^2 = \left(\frac{-3}{x}\right)^2$

$x^2 + y^2 = \frac{9}{x^2}$

$x^4 + x^2y^2 = 9$

E. $\left(\frac{r}{3\tan\theta} = \sin\theta\right)$

$r \cdot \frac{1}{\frac{3\sin}{\cos}} = \sin$

$\frac{r\cos\theta}{3\sin\theta} = \sin\theta$

$r\cos\theta = 3\sin^2\theta$

$x = 3\sin^2\theta$

Converting from Rectangular Form to Polar Form

EXAMPLE 5 Convert each rectangular equation to polar form.

A. $x^2 + y^2 = 1$

$r^2 = 1$

$r = \pm 1$

B. $y = 2x + 1$

$r\sin\theta = 2r\cos\theta + 1$

$r\sin\theta - 2r\cos\theta = 1$

$r(\sin\theta - 2\cos\theta) = 1$

$r = \frac{1}{\sin\theta - 2\cos\theta}$

C. $y = \frac{3}{x}$

$\left(r\sin\theta = \frac{3}{r\cos\theta}\right) r\cos\theta$

$\frac{r^2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{3}{\sin\theta\cos\theta}$

$\sqrt{r^2} = \sqrt{\frac{3}{\sin\theta\cos\theta}}$

$r = \pm\sqrt{\frac{3}{\sin\theta\cos\theta}}$

D. $(x-3)^2 + (y-2)^2 = 13$

$x^2 - 6x + 9 + y^2 - 4x + 4 = 13$

$x^2 + y^2 - 6x - 4y = 0$

$r^2 - 6r\cos\theta - 4r\sin\theta = 0$

$r(r - 6\cos\theta - 4\sin\theta) = 0$

$\cancel{r=0}$ $r = 6\cos\theta + 4\sin\theta$

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1. a) $(4, 60°)$
$(-4, 240°)$
$(4, -300°)$

b) $(-5, 315°)$
$(5, 135°)$
$(-5, -45°)$

c) $(2, -90°)$
$(2, 270°)$
$(-2, 90°)$

d) $(1, \frac{5\pi}{6})$
$(-1, \frac{11\pi}{6})$
$(1, -\frac{7\pi}{6})$

e) $(-8, \frac{\pi}{6})$
$(-8, -\frac{11\pi}{6})$
$(8, \frac{7\pi}{6})$

f) $(-\frac{3}{2}, -\frac{5\pi}{3})$
$(-\frac{3}{2}, \frac{\pi}{3})$
$(\frac{3}{2}, \frac{4\pi}{3})$

2. a) $(6, 90°)$
$(6\cos 90, 6\sin 90)$
$(0, 6)$

b) $(5, 60°)$
$(5\cos 60, 5\sin 60)$
$(\frac{5}{2}, \frac{5\sqrt{3}}{2})$

c) $(10, 225°)$
$(10\cos 225, 10\sin 225)$
$(-5\sqrt{2}, -5\sqrt{2})$

d) $(5, \pi)$
$(5\cos\pi, 5\sin\pi)$
$(-5, 0)$

e) $(2\sqrt{3}, \frac{\pi}{6})$
$(2\sqrt{3}\cos\frac{\pi}{6}, 2\sqrt{3}\sin\frac{\pi}{6})$
$(3, \sqrt{3})$

f) $(\frac{5}{2}, \frac{5\pi}{3})$
$(\frac{5}{2}\cos\frac{5\pi}{3}, \frac{5}{2}\sin\frac{5\pi}{3})$
$(\frac{5}{4}, -\frac{5\sqrt{3}}{4})$

3. a) $(-5, -5)$
$r = 5\sqrt{2}$
$\theta = 225°$
$(5\sqrt{2}, 225)$

b) $(0, -2)$
$r = 2$
$\theta = 270$
$(2, 270)$

c) $(1, -\sqrt{3})$
$r = 2$
$\theta = \tan^{-1}(\frac{-\sqrt{3}}{1})$
$\theta = -\frac{\pi}{3}$
$(2, -\frac{\pi}{3})$

d) $(-7, 0)$
$r = 7$
$\theta = 180$
$(7, 180)$

e) $(5, 12)$
$r = 13$
$\theta = \tan^{-1}(\frac{12}{5})$
$\theta = 67.4°$
$(13, 67.4°)$

f) $(6, -3)$
$r = 3\sqrt{5}$
$\theta = \tan^{-1}(\frac{-3}{6})$
$\theta = -26.6°$
$(3\sqrt{5}, -26.6°)$

4. a) $x^2 + y^2 = 25$
$r^2 = 25$
$r = \pm 5$

b) $(x+2)^2 + y^2 = 4$
$x^2 + 4x + 4 + y^2 = 4$
$x^2 + y^2 + 4x = 0$
$r^2 + 4r\cos\theta = 0$
$r = 0$ $\boxed{r = -4\cos\theta}$

(4) (c) $y=3$

$r\sin\theta=3$

$\boxed{r=\frac{3}{\sin\theta}}$

(d) $x=3$

$r\cos\theta=3$

$\boxed{r=\frac{3}{\cos\theta}}$

(e) $xy=1$

$r\cos\theta\, r\sin\theta=1$

$r^2=\frac{1}{\cos\theta\sin\theta}$

$\boxed{r=\pm\sqrt{\sec\theta\csc\theta}}$

(f) $2x-3y-2=0$

$2r\cos\theta-3r\sin\theta=2$

$r(2\cos\theta-3\sin\theta)=2$

$\boxed{r=\frac{2}{2\cos\theta-3\sin\theta}}$

(5) (a) $r=2$

$r^2=4$

$\boxed{x^2+y^2=4}$

(b) $\theta=\frac{2\pi}{3}$

$\tan\theta=\tan\frac{2\pi}{3}$

$\frac{y}{x}=-\sqrt{3}$

$\boxed{y=-x\sqrt{3}}$

(c) $r=4\sec\theta$

$r=\frac{4}{\cos\theta}$

$r\cos\theta=4$

$\boxed{x=4}$

(d) $r=-2\csc\theta$

$r=\frac{-2}{\sin\theta}$

$r\sin\theta=-2$

$\boxed{y=-2}$

(e) $\left(r=\frac{12}{3\sin\theta-4\cos\theta}\right)(3\sin\theta-4\cos\theta)$

$3r\sin\theta-4r\cos\theta=12$

$3y-4x=12$

$y=\frac{4x}{3}+\frac{12}{3}$

$\boxed{y=\frac{4}{3}x+4}$

(f) $r=\frac{3}{1+\sin\theta}$

$r+r\sin\theta=3$

$r+y=3$

$(r)^2=(3-y)^2$

$r^2=9-6y+y^2$

$x^2+y^2=9-6y+y^2$

$x^2-9=-6y$

$y=-\frac{1}{6}(x^2-9)$