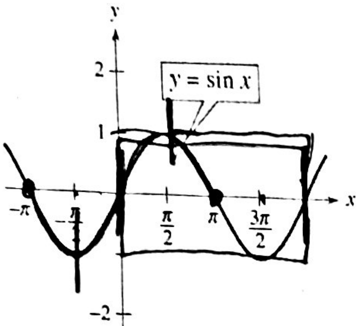
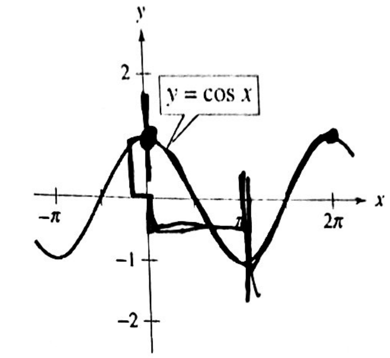
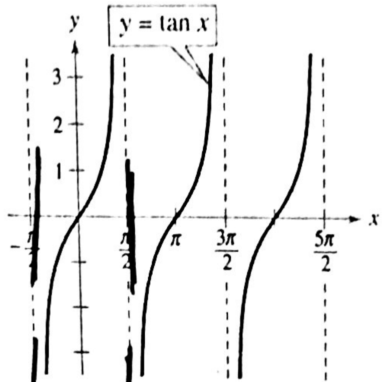
A function will only have an inverse that is a function given that it is one to one.
Since we know what the graphs of sine, cosine, and tangent look like, it is clear that they are not one-to-one. However, if you restrict the domain of each function to an interval, then the restricted function IS one-to-one! **The inverse function is the inverse of the restricted portion of the function.**



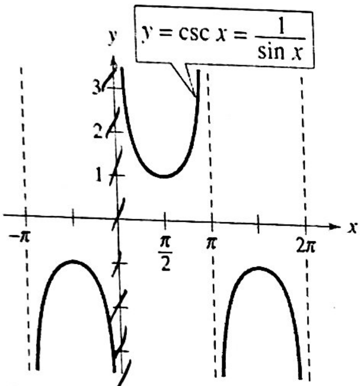
DOMAIN: $(-\infty, \infty)$
RANGE: $[-1, 1]$
PERIOD: 2π



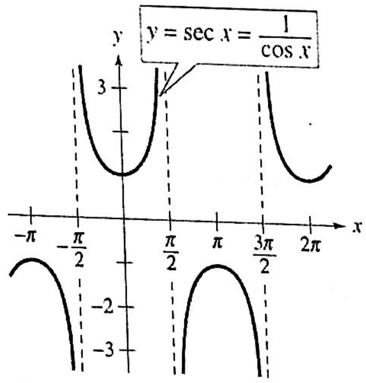
DOMAIN: $(-\infty, \infty)$
RANGE: $[-1, 1]$
PERIOD: 2π



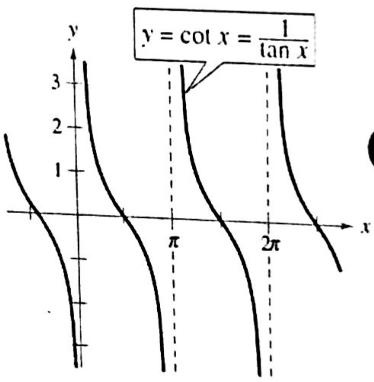
DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, \infty)$
PERIOD: π



DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
PERIOD: 2π



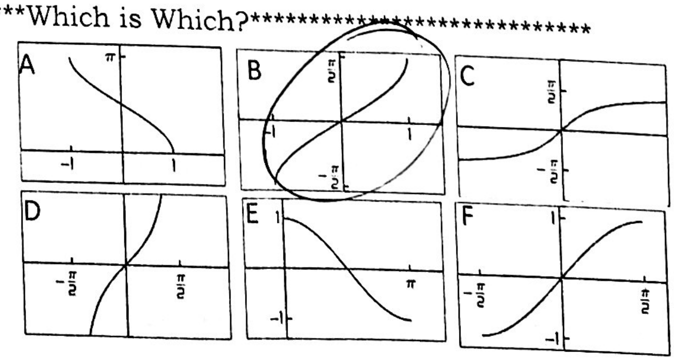
DOMAIN: ALL $x \neq \frac{\pi}{2} + n\pi$
RANGE: $(-\infty, -1] \cup [1, \infty)$
PERIOD: 2π



DOMAIN: ALL $x \neq n\pi$
RANGE: $(-\infty, \infty)$
PERIOD: π

*****************************Which is Which?*****************************

1) $y = \sin x$ F
2) $y = \cos x$ E
3) $y = \tan x$ D
4) $y = \arcsin x$ $\sin^{-1}(x)$ B
5) $y = \arccos x$ A
6) $y = \arctan x$ C



THE ARCSINE FUNCTION

DEFINITION------The unique angle y in the interval $[-\pi/2, \pi/2]$ such that-- $\sin(y) = x$ -- is the inverse sine (or arcsine) of x. Denoted $\sin^{-1}x$ or *arcsin x*. The domain of $y = \sin^{-1}x$ is [-1, 1] and the range is [~~$-\infty$~~, ~~∞~~]

$-\frac{\pi}{2}, \frac{\pi}{2}$

$\boxed{30° = \frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)}$

Ex1) Find the exact value of each expression without a calculator:

(a) $\sin^{-1}\left(\frac{1}{2}\right)$ 30° $\frac{\pi}{6}$

(b) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ -60 $-\frac{\pi}{3}$

(c) $\sin^{-1}\left(\frac{\pi}{2}\right)$ No Sol.

(d) $\sin^{-1}\left(\sin\left(\frac{\pi}{9}\right)\right)$ $\frac{\pi}{9}$

(e) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$ $\sin^{-1}\left(\frac{1}{2}\right)$ $\frac{\pi}{6}$

Ex2) Use a calculator to evaluate the following values:

(a) $\sin^{-1}(-0.81)$ $-.9$

(b) $\sin^{-1}(\sin(3.49\pi))$ -1.5

*What mode should be used? Radians

*How do you know? $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

THE ARCCOSINE & ARCTANGENT FUNCTIONS

DEFINITION------The unique angle y in the interval $[0, \pi]$ such that--$\cos(y) = x$--is the inverse cosine (or arccosine) of x. Denoted $\cos^{-1}x$ or *arccos x*. The domain of $y = \cos^{-1}x$ is $[-1, 1]$ and the range is $[0, \pi]$ $[0, \pi]$

DEFINITION------The unique angle y in the interval $(-\pi/2, \pi/2)$ such that--$\tan(y)=x$-- is the inverse tangent (or arctangent) of x. Denoted $\tan^{-1}x$ or *arctan x*. The domain of $y = \tan^{-1}x$ is $(-\infty, \infty)$ and the range is $(-\pi/2, \pi/2)$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ex3) Find the exact value of the following expressions without a calculator:

(a) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ 45 → $\frac{\pi}{4}$ $\frac{3\pi}{4}$

(b) $\tan^{-1}(\sqrt{3})$ $\frac{y}{x}$ $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\boxed{\frac{\pi}{3}}$

(c) $\cos^{-1}(\cos(-1.1)) = 1.1$ ~~2.04~~

Precalculus

Name: ____________________

$30^\circ = \frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Practice With Inverse Trig Functions

Find the exact value of the expression whenever it is defined.

1. a. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $-\frac{\pi}{4}$ b. $\cos^{-1}\left(-\frac{1}{2}\right)$ $\frac{2\pi}{3}$ c. $\tan^{-1}\left(-\sqrt{3}\right)$ $\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $-\frac{\pi}{3}$

2. a. $\sin^{-1}\left(-\frac{1}{2}\right)$ $-\frac{\pi}{6}$ b. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $\frac{3\pi}{4}$ c. $\tan^{-1}(-1)$ $-\frac{\pi}{4}$

3. a. $\arcsin\frac{\sqrt{3}}{2}$ $\frac{\pi}{3}$ b. $\arccos\frac{\sqrt{2}}{2}$ $\frac{\pi}{4}$ c. $\arctan\frac{1}{\sqrt{3}}$ $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\frac{\pi}{6}$

4. a. $\arcsin 0$ 0 b. $\arccos(-1)$ π c. $\arctan 0$ $\frac{0}{1}$ 0

5. a. $\sin^{-1}\left(\frac{\pi}{3}\right)$ $\emptyset$ b. $\cos^{-1}\left(\frac{\pi}{2}\right)$ $\emptyset$ c. $\tan^{-1}(1)$ $\frac{\pi}{4}$

6. a. $\arcsin\left(\frac{\pi}{2}\right)$ $\emptyset$ b. $\arccos\left(\frac{\pi}{3}\right)$ $\emptyset$ c. $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{1}{\sqrt{3}}$ $-\frac{\pi}{6}$

7. a. $\sin\left[\arcsin\left(-\frac{3}{10}\right)\right]$ $-\frac{3}{10}$ b. $\cos\left[\arccos\frac{1}{2}\right]$ $\frac{1}{2}$ c. $\tan(\arctan 14)$ 14

8. a. $\sin\left(\sin^{-1}\frac{2}{3}\right)$ $\frac{2}{3}$ b. $\cos\left[\cos^{-1}\left(-\frac{1}{5}\right)\right]$ $-\frac{1}{5}$ c. $\tan\left[\tan^{-1}(-9)\right]$ -9

$30 = \frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

9. a. $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$ $\frac{\pi}{3}$ b. $\cos^{-1}\left[\cos\left(\frac{5\pi}{6}\right)\right]$ $\frac{5\pi}{6}$ c. $\tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right]$ $-\frac{\pi}{6}$

10. a. $\arcsin\left[\sin\left(-\frac{\pi}{2}\right)\right]$ $-\frac{\pi}{2}$ b. $\arccos(\cos 0)$ 0 c. $\arctan\left(\tan\frac{\pi}{4}\right)$ $\frac{\pi}{4}$

11. a. $\arcsin\left(\sin\frac{5\pi}{4}\right)$ $-\frac{\pi}{4}$ b. $\arccos\left(\cos\frac{5\pi}{4}\right)$ $\frac{3\pi}{4}$ c. $\arctan\left(\tan\frac{7\pi}{4}\right)$ $-\frac{\pi}{4}$

12. a. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ $\frac{\pi}{3}$ b. $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$ $\frac{2\pi}{3}$ c. $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ $\frac{\pi}{6}$

13. a. $\sin\left[\cos^{-1}\left(-\frac{1}{2}\right)\right]$ $\sin\left(\frac{2\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$ b. $\cos\left(\tan^{-1} 1\right)$ $\cos\frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$ c. $\tan\left[\sin^{-1}(-1)\right]$ $\tan -\frac{\pi}{2}$ Undefined

14. a. $\sin\left[\tan^{-1}\sqrt{3}\right]$ $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\sin\frac{\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$ b. $\cos\left[\sin^{-1} 1\right]$ $\cos\frac{\pi}{2} = 0$ c. $\tan\left(\cos^{-1} 0\right)$ $\tan\frac{\pi}{2}$ Undefined

15. a. $\cot\left(\sin^{-1}\frac{2}{3}\right)$ $\boxed{\frac{\sqrt{5}}{2}}$ b. $\sec\left[\tan^{-1}\left(-\frac{3}{5}\right)\right]$ $\frac{1}{\cos} = \frac{1}{\frac{4}{5}} = \boxed{\frac{5}{4}}$ c. $\csc\left[\cos^{-1}\left(-\frac{1}{4}\right)\right]$ $\frac{1}{\sin} = \frac{1}{\frac{\sqrt{15}}{4}} = \boxed{\frac{4}{\sqrt{15}}}$

16. a. $\cot\left[\sin^{-1}\left(-\frac{2}{5}\right)\right]$ $\boxed{\frac{\sqrt{21}}{-2}}$ b. $\sec\left[\tan^{-1}\left(\frac{7}{4}\right)\right]$ $\frac{1}{\cos}$ $\frac{1}{\frac{4}{\sqrt{65}}} = \boxed{\frac{\sqrt{65}}{4}}$ c. $\csc\left[\cos^{-1}\left(\frac{1}{5}\right)\right]$ $\sqrt{24}$ or $2\sqrt{6}$ $\csc = \frac{1}{\sin} = \frac{1}{\frac{\sqrt{24}}{5}} = \boxed{\frac{5}{\sqrt{24}}}$

Notes—Graphing Sinusoids

Definition: A function is a sinusoid if it can be written in the form → $f(x) = a \cdot \sin(bx + c) + d$
(where a, b, c, and d are constants and neither a nor b is 0) **OR** $f(x) = a \cdot \cos(bx + c) + d$

Definition: The amplitude of a sinusoid of the form form → $f(x) = a \cdot \sin(bx + c) + d$ **OR** $f(x) = a \cdot \cos(bx + c) + d$ is $|a|$. Graphically, the amplitude is half the height of the wave.

Definition: The period of a sinusoid of the form form → $f(x) = a \cdot \sin(bx + c) + d$ **OR** $f(x) = a \cdot \cos(bx + c) + d$ is $\frac{b}{2\pi}$. Graphically, the period is the length of one full wave.

Definition: The frequency of a sinusoid of the form form → $f(x) = a \cdot \sin(bx + c) + d$ **OR** $f(x) = a \cdot \cos(bx + c) + d$ is $\frac{2\pi}{b}$. Graphically, the frequency is the number of waves that occur every 2π.

Things to keep in mind . . .

- The basic graphs of sine and cosine have a period of 2π.
- Changes in amplitude and period as well as phase shifts are nothing more than transformations you've seen before; they have just been given new names for trig functions.
  - Changes in amplitude are vertical stretches or shrinks/compressions
  - Changes in period are horizontal stretches or shrinks/compressions
  - Phase shifts are horizontal (left or right) shifts
  - These graphs can also be shifted vertically

Graphs of Sinusoids

The graphs of $y = a\sin(b(x - h)) + k$ and $y = a\cos(b(x - h)) + k$ (where $a \neq 0$ and $b \neq 0$) have the following characteristics:

amplitude $= |a|$;

period $= \dfrac{2\pi}{|b|}$;

frequency $= \dfrac{|b|}{2\pi}$.

When compared to the graphs of $y = a\sin bx$ and $y = a\cos bx$, respectively, they also have the following characteristics:

a phase shift of h;

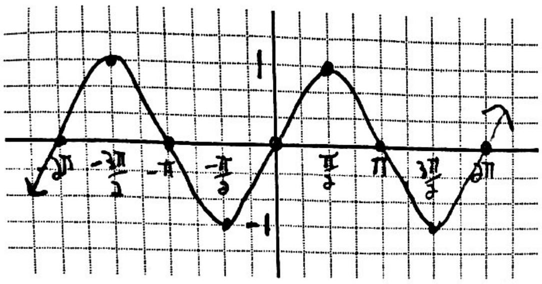
a vertical translation of k.

For the graphs of $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$

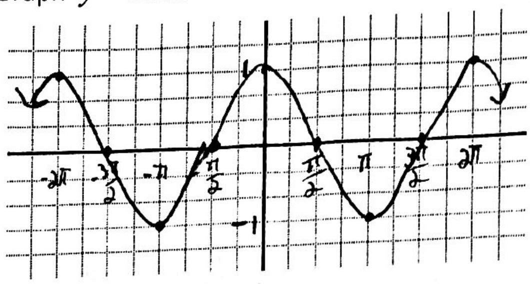
- Amplitude $= |A|$
- Period $= \dfrac{2\pi}{B}$
- Phase Shift $= \dfrac{C}{B}$
- Vertical Shift $= D$
- Distance Between Key Points* $= \left(\dfrac{1}{4}\right) \cdot (\text{period})$

**Key Points* are the points that are at the top or bottom of the graph, or the points on the center-line of the graph

Graph $y = \sin x$



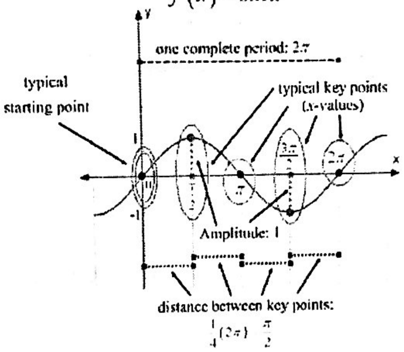
Graph $y = \cos x$



Here are examples of a single change to each of these elements for the basic sine graph.

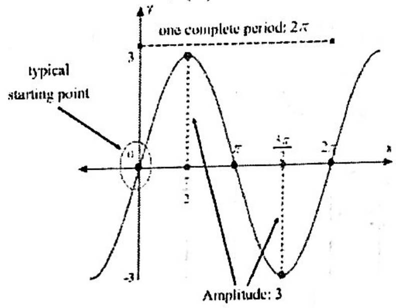
Basic Sine Graph

$f(x) = \sin x$



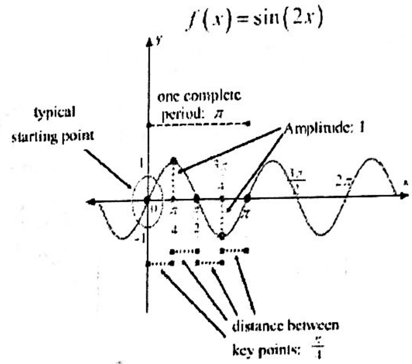
Change in Amplitude

$f(x) = 3\sin x$



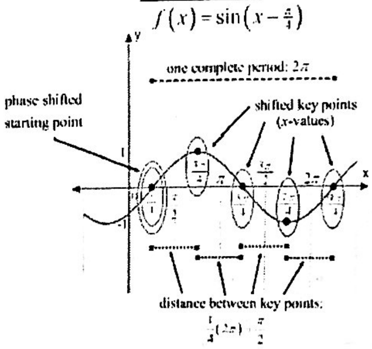
Change in Period

$f(x) = \sin(2x)$



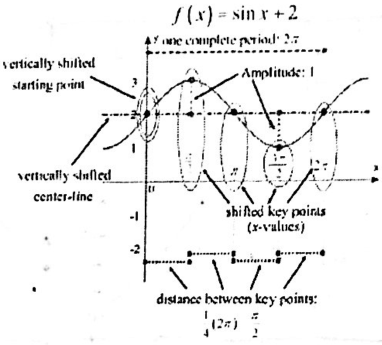
Phase Shift

$f(x) = \sin\left(x - \frac{\pi}{4}\right)$



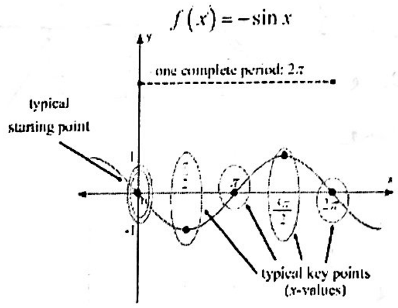
Vertical Shift

$f(x) = \sin x + 2$



Reflection

$f(x) = -\sin x$



Example 1 Find the amplitude of each of the following sinusoids & then use the language of transformations to describe how the graphs of b and c are related to a.

a) $f(x) = \cos x$ amp = 1

b) $y = \frac{1}{2}\cos x$ amp = $\frac{1}{2}$

c) $y = -3\cos x$ amp = 3

Example 2 Find the period of each of the following sinusoids & then use the language of transformations to describe how the graphs of b and c are related to a.

a) $f(x) = \sin x$ pd = 2π

b) $y = 3\sin(-2x)$ pd = $\frac{2\pi}{2} = \pi$

c) $y = -2\sin\left(\frac{x}{3}\right)$ pd = $\frac{2\pi}{\frac{1}{3}} = 6\pi$

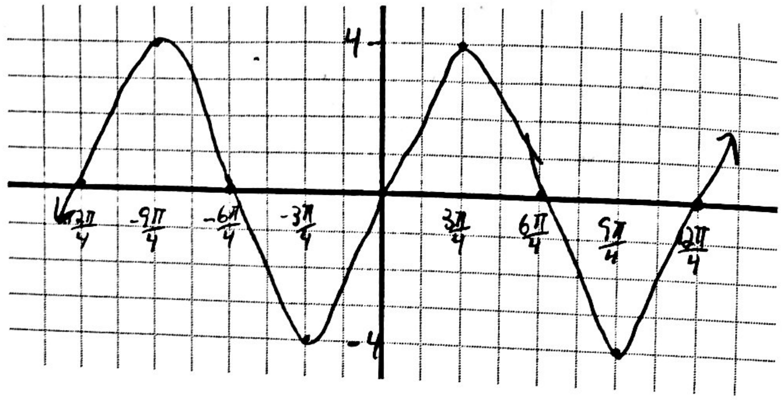
Example 3 Find the amplitude, period, and frequency of the function $f(x) = 4\sin\left(\frac{2x}{3}\right)$. Sketch the graph.

$f(x) = 4\sin\frac{2}{3}x$

Amp = 4

$Pd = \frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2} = 3\pi$

* Usually divide Period by 4, to see what to count by.



Example 4 Find the amplitude, period, phase shift, vertical shift, and any reflection.

$y = -2\cos 4\left(x + \frac{\pi}{4}\right)$

Amp : 2

Pd. : $\frac{2\pi}{4} = \frac{\pi}{2}$

Ph Sh.: ← $\frac{\pi}{4}$

Ver Sh.: None

reflect over x-axis

Example 5 Find the amplitude, period, phase shift, vertical shift, and any reflection. Then graph one complete period.

A. $y = 3\sin(6\pi x)$

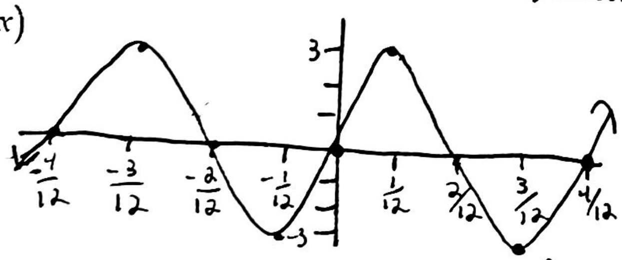
Amp: 3

Pd.: $\frac{2\pi}{6\pi} = \frac{1}{3}$

PS: None

VS: None

Ref: None



B. $y = -2\cos(x) + 3$

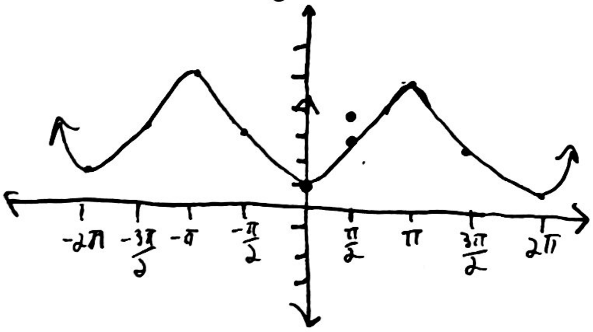
Amp: 2

Pd : 2π

PS : None

VS : ↑3

Ref : reflect over x-axis



C. $y = -3\sin\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) - 2$: $y = -3\sin\frac{\pi}{2}\left(x + \frac{1}{2}\right) - 2$

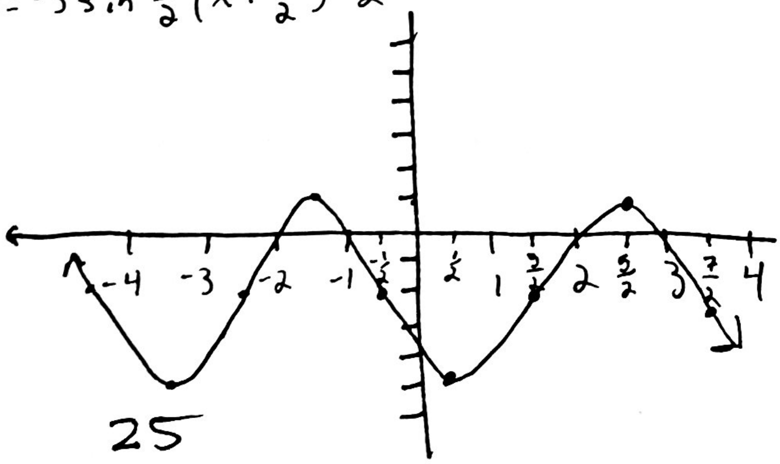
Amp: 3

Pd : $\frac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \frac{2}{\pi} = 4$

PS : ← $\frac{1}{2}$

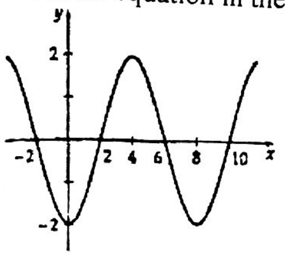
VS: ↓ 2

Ref : reflect over x-axis



Example 6 Write an equation in the form y = Asin(Bx) or y = Acos(Bx) for each graph.

A.



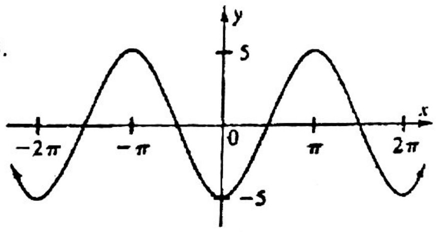
$y = -2\cos \frac{\pi}{4} x$

$Pd: 8 \quad \frac{2\pi}{b} : 8$

$b = \frac{2\pi}{8}$

$b = \frac{\pi}{4}$

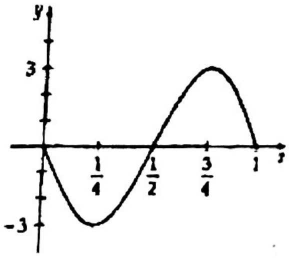
B.



$y = -5\cos x$

$Pd = 2\pi \quad b = 1$

C.



$y = -3\sin 2\pi x$

$Pd = 1 \quad \frac{2\pi}{b} = 1$

$b = 2\pi$

Example 7 Write an equation for each description or graph.

A. sine function; amplitude = 2, period = $\frac{\pi}{3}$, shifted down 1 unit

$y = 2\sin(6x) - 1$

$\frac{\pi}{3} = \frac{2\pi}{b}$

$b = 6$

B. cosine function; amplitude = 4, period = 3, shifted right $\frac{\pi}{8}$ units, shifted up 2 units

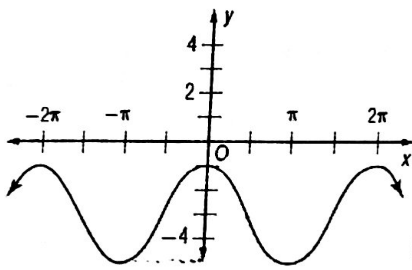
$y = 4\cos \frac{2\pi}{3}\left(x - \frac{\pi}{8}\right) + 2$

$Pd: 3$

$\frac{2\pi}{b} = 3$

$b = \frac{2\pi}{3}$

C. sine equation



$Pd = 2\pi \quad b = 1$

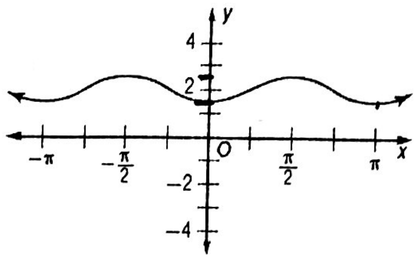
$Amp = 2$

$P.S. : \leftarrow \frac{\pi}{2}$

$V.S. \downarrow 3$

$y = 2\sin\left(x + \frac{\pi}{2}\right) - 3$

D. cosine equation



$Amp = \frac{1}{2}$

$Pd. : \pi$

$P.S. :$ None

$V.S. : \uparrow 2$

$y = -\frac{1}{2}\cos(2x) + 2$

$\frac{2\pi}{b} = \pi \quad b = 2$