

* In Trigonometry we look at an angle in terms of a rotating ray. The beginning position of the ray is called the initial side of the angle.

* The ray is rotated about its end point called the vertex and the final position of the ray is called the terminal side of the angle.

* The measure of an angle is a number that describes the amount of rotation from the initial side to the terminal side of the angle.

Positive angles are generated by counter clockwise rotations

Negative angles are generated by clockwise rotations

***NOTE: Typically angles are drawn in STANDARD POSITION with vertex at the origin & initial side on the positive x-axis.

* Because it is possible for two angles to have the same initial side and terminal side but different angle measures we refer to these angles as coterminal angles.

Ex 1) Find 2 positive and 2 negative angles that are co-terminal with the given angle.

a) $30^\circ \pm 360$

$\begin{matrix} + \\ 390^\circ \\ 750^\circ \end{matrix} \quad \begin{matrix} - \\ -330^\circ \\ -690^\circ \end{matrix}$

b) $-150^\circ \pm 360$

$\begin{matrix} + \\ 210^\circ \\ 570^\circ \end{matrix} \quad \begin{matrix} - \\ -510^\circ \\ -870^\circ \end{matrix}$

c) $\frac{2\pi}{3} \pm \frac{6\pi}{3}$

$\begin{matrix} + \\ \frac{8\pi}{3} \\ \frac{14\pi}{3} \end{matrix} \quad \begin{matrix} - \\ -\frac{4\pi}{3} \\ -\frac{10\pi}{3} \end{matrix}$

d) $300^\circ \pm 360$

$\begin{matrix} + \\ 660^\circ \\ 1020^\circ \end{matrix} \quad \begin{matrix} - \\ -60^\circ \\ -420^\circ \end{matrix}$

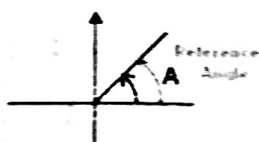
e) $-210^\circ \pm 360$

$\begin{matrix} + \\ 150^\circ \\ 510^\circ \end{matrix} \quad \begin{matrix} - \\ -570^\circ \\ -930^\circ \end{matrix}$

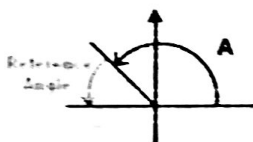
f) $\frac{5\pi}{4} \pm \frac{8\pi}{4}$

$\begin{matrix} + \\ \frac{13\pi}{4} \\ \frac{21\pi}{4} \end{matrix} \quad \begin{matrix} - \\ -\frac{3\pi}{4} \\ -\frac{11\pi}{4} \end{matrix}$

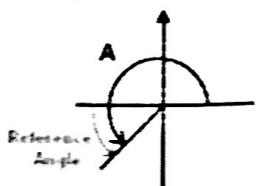
reference angle – the angle that the given angle makes with the x-axis. Regardless of where the angle ends (that is, regardless of the location of the terminal side of the angle), the reference angle measures the closest distance of that terminal side to the x-axis.



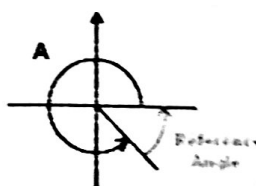
1st Quadrant



2nd Quadrant

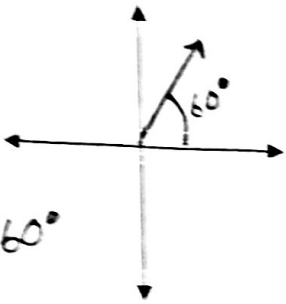
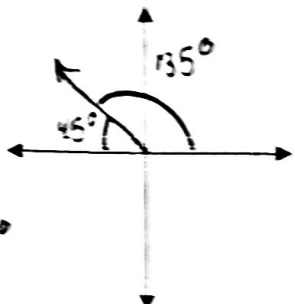
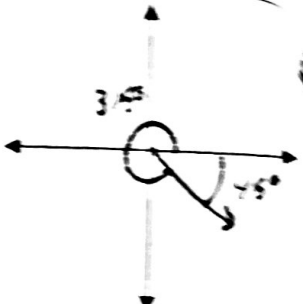
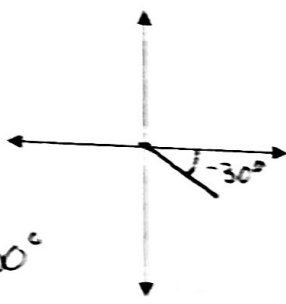
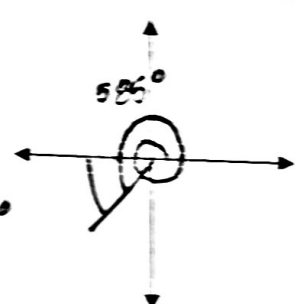
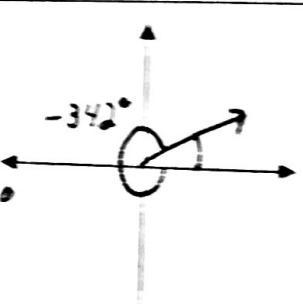


3rd Quadrant



4th Quadrant

Ex 2) Draw each angle in standard position, and find its reference angle.

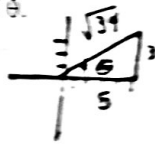
$\theta = 60^\circ$  ref: 60°	$\theta = 135^\circ$  ref: 45°	$\theta = 315^\circ$  ref: 45°
$\theta = -30^\circ$  ref: 30°	$\theta = 585^\circ$  ref: 45°	$\theta = -342^\circ$  ref: 18°

Ex 3) Let θ be the acute angle in standard position whose terminal side contains (5, 3). Find the 6 trig functions of θ .

$$\sin \theta = \frac{3}{\sqrt{34}} \quad \csc \theta = \frac{\sqrt{34}}{3}$$

$$\cos \theta = \frac{5}{\sqrt{34}} \quad \sec \theta = \frac{\sqrt{34}}{5}$$

$$\tan \theta = \frac{3}{5} \quad \cot \theta = \frac{5}{3}$$

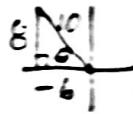


Ex 4) Find the six trig functions of an angle whose terminal side passes through (-6, 8).

$$\sin \theta = \frac{8}{10} \quad \csc \theta = \frac{10}{8}$$

$$\cos \theta = \frac{-6}{10} \quad \sec \theta = \frac{10}{-6}$$

$$\tan \theta = \frac{8}{-6} \quad \cot \theta = \frac{-6}{8}$$



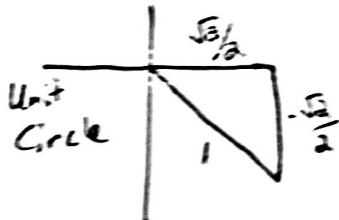
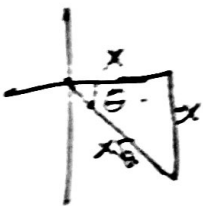
Ex 5) Find the six trig functions of 315°



$$\sin \theta = \frac{-2}{\sqrt{5}} = -\frac{2}{\sqrt{5}} \quad \csc \theta = -\sqrt{5}$$

$$\cos \theta = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}} \quad \sec \theta = \sqrt{5}$$

$$\tan \theta = -1 \quad \cot \theta = -1$$



Ex 6) Find each of the following:

a) $\sin(-210^\circ) = \frac{1}{2}$

b) $\tan(5\pi/3) = -\frac{\sqrt{3}}{1} = -\sqrt{3}$

c) $\sec(-3\pi/4) = \frac{1}{\cos(-3\pi/4)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$ or $-\sqrt{2}$

* Angles whose terminal sides lie along one of the coordinate axes are called Quadrantal angles.

Ex 7) Find each of the following, if it exists:

a) $\sin(-270^\circ)$ |

b) $\tan 3\pi$
 $(-1, 0) \quad \frac{0}{-1} = 0$

$\sec = \frac{1}{\cos}$
 c) $\sec(11\pi/2) = \frac{1}{0} = \text{Does Not Exist}$

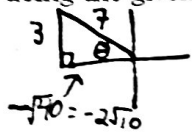
d) $\cos(180^\circ)$ -|

e) $\cot(0^\circ)$: $\frac{1}{0}$ Does Not Exist
 $\frac{1}{\tan}$ or $\frac{\cos}{\sin}$
 $(1, 0)$

f) $\csc(-\pi/2) = \frac{1}{-1} = -1$
 $\frac{1}{\sin} : (0, -1)$

Ex 8) Find $\cos\theta$ and $\tan\theta$ using the given information to construct a reference angle.

a) $\sin\theta = 3/7$ I + II
 $\tan\theta < 0$ II + IV

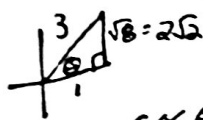


$\cos\theta = -\frac{2\sqrt{10}}{7}$

$\tan\theta = \frac{3}{-2\sqrt{10}}$
 $-\frac{3\sqrt{10}}{20}$

b) $\sec\theta = 3$
 $\sin\theta > 0$

$\cos\theta = \frac{1}{3}$ (I + IV)
 $\sin\theta > 0$ (I + II)



$\cos\theta = \frac{1}{3}$

$\tan\theta = \frac{2\sqrt{2}}{3} = 2\sqrt{2}$

c) $\cot\theta$ is undefined
 $\sec\theta < 0$ I + IV



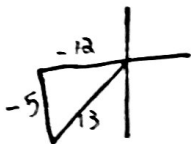
$\cos\theta = 1$

$\tan\theta = \frac{0}{1} = 0$

$\frac{x}{y} \quad 0^\circ \text{ or } 180^\circ$

Ex 9) Find $\sin\theta$ and $\cot\theta$ using the given information to construct a reference angle.

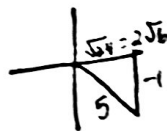
d) $\cos\theta = -12/13$ II + III
 $\sin\theta < 0$ III + IV



$\sin\theta = -5/13$

$\cot\theta = \frac{2}{-5} = -\frac{12}{5} = \frac{12}{5}$

e) $\csc\theta = -5$ III + IV
 $\tan\theta < 0$ II + IV



$\sin\theta = -\frac{1}{5}$

$\cot\theta = \frac{2\sqrt{6}}{-1} = -2\sqrt{6}$

f) $\tan\theta$ is undefined
 $\sin\theta > 0$ I + II



$\sin\theta = 1$

$\cot\theta = 0$

$\frac{x}{y} \quad 90^\circ \text{ or } 270^\circ$

Precalculus Unit 5

Homework—Trig Functions of Any Angle

$$30^\circ = \frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

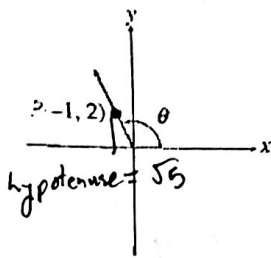
In exercises 1-2, identify the angle that is not coterminal with all the others.

1. $150^\circ, 510^\circ, -210^\circ, \textcircled{450^\circ}, 870^\circ$

2. $\frac{5\pi}{3}, \textcircled{-\frac{5\pi}{3}}, \frac{11\pi}{3}, -\frac{7\pi}{3}, \frac{365\pi}{3}$

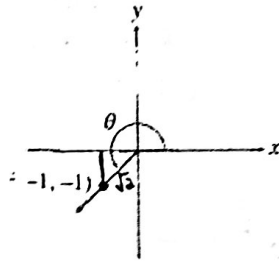
In exercises 3-4, evaluate the six trig functions of the angle θ .

3.



$$\begin{aligned} \sin \theta &= \frac{2}{\sqrt{5}} \\ \cos \theta &= -\frac{1}{\sqrt{5}} \\ \tan \theta &= \frac{2}{-1} = -2 \\ \csc \theta &= \frac{\sqrt{5}}{2} \\ \sec \theta &= -\sqrt{5} \\ \cot \theta &= -\frac{1}{2} \end{aligned}$$

4.



$$\begin{aligned} \sin \theta &= -\frac{1}{\sqrt{2}} \\ \cos \theta &= -\frac{1}{\sqrt{2}} \\ \tan \theta &= 1 \\ \csc \theta &= -\sqrt{2} \\ \sec \theta &= -\sqrt{2} \\ \cot \theta &= 1 \end{aligned}$$

In exercises 5-7, point P is on the terminal side of angle θ . Evaluate the six trig functions for θ . If the function is undefined, write "undefined".

5. P(3, 4)

6. P(0, 5)

7. P(5, -2)

In exercises 8-11, state the sign (+ or -) of (a) $\sin t$, (b) $\cos t$, and (c) $\tan t$ for values of t in the interval given.

8. $\left(0, \frac{\pi}{2}\right)$

9. $\left(\frac{\pi}{2}, \pi\right)$

10. $\left(\pi, \frac{3\pi}{2}\right)$

11. $\left(\frac{3\pi}{2}, 2\pi\right)$

In exercises 12-15, determine the sign (+ or -) of the given value without the use of a calculator.

12. $\cos 143^\circ$

13. $\tan 192^\circ$

14. $\cos \frac{7\pi}{8}$

15. $\tan \frac{4\pi}{5}$

In exercises 16-17, choose the point on the terminal side of θ .

16. $\theta = 45^\circ$

A. (2, 2)

B. $(1, \sqrt{3})$

C. $(\sqrt{3}, 1)$

17. $\theta = \frac{7\pi}{6}$

A. $(-\sqrt{3}, -1)$

B. $(-1, \sqrt{3})$

C. $(-\sqrt{3}, 1)$

In exercises 18-29, evaluate without using a calculator by using ratios in a reference triangle.

18. $\cos 120^\circ$

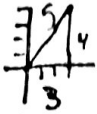
19. $\tan 300^\circ$


20. $\sec \frac{\pi}{3}$

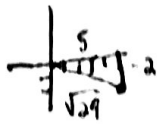
21. $\csc \frac{3\pi}{4}$

22. $\sin \frac{13\pi}{6}$

Unit 5 Pg 17 HW

(5) 
 $\sin \theta = \frac{4}{5}$ $\csc \theta = \frac{5}{4}$
 $\cos \theta = \frac{3}{5}$ $\sec \theta = \frac{5}{3}$
 $\tan \theta = \frac{4}{3}$ $\cot \theta = \frac{3}{4}$

(6) 
 $\sin \theta = 5$ $\csc \theta = \frac{1}{5}$
 $\cos \theta = 0$ $\sec \theta = \text{Und}$
 $\tan \theta = \text{Und}$ $\cot \theta = 0$

(7) 
 $\sin \theta = -\frac{2}{\sqrt{29}}$ $\csc \theta = \frac{\sqrt{29}}{-2}$
 $\cos \theta = \frac{5}{\sqrt{29}}$ $\sec \theta = \frac{\sqrt{29}}{5}$
 $\tan \theta = -\frac{2}{5}$ $\cot \theta = \frac{5}{-2}$

(8) $(0, \frac{\pi}{2})$

(9) $(\frac{\pi}{2}, \pi)$

(10) $(\pi, \frac{3\pi}{2})$

(11) $(\frac{3\pi}{2}, 2\pi)$

- (a) $\sin t = +$
- (b) $\cos t = +$
- (c) $\tan t = +$

- (a) $\sin t = +$
- (b) $\cos t = -$
- (c) $\tan t = -$

- (a) $\sin t = -$
- (b) $\cos t = -$
- (c) $\tan t = +$

- (a) $\sin t = -$
- (b) $\cos t = +$
- (c) $\tan t = -$

(12) $\cos 143 = -$

(16) $\theta = 45^\circ$ A

(18) $\cos 120 = -\frac{1}{2}$

(13) $\tan 192 = +$

(17) $\theta = \frac{7\pi}{6}$ A

(19) $\tan 300 = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

(14) $\cos \frac{7\pi}{8} = -$

(20) $\sec \frac{\pi}{3} = \frac{1}{\cos \frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

(15) $\tan \frac{4\pi}{5} = -$

(21) $\csc \frac{3\pi}{4} = \frac{1}{\sin \frac{3\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$ or $\sqrt{2}$

(22) $\sin \frac{13\pi}{6} = \frac{1}{2}$

(23) $\cos \frac{7\pi}{3} = \frac{1}{2}$

(24) $\tan(-\frac{15\pi}{4}) = 1$

(25) $\cot \frac{13\pi}{4} = 1$

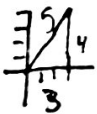
(26) $\cos \frac{23\pi}{6} = \frac{\sqrt{3}}{2}$


(27) $\cos \frac{17\pi}{4} = \frac{\sqrt{2}}{2}$

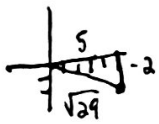
(28) $\sin \frac{11\pi}{3} = -\frac{\sqrt{3}}{2}$

(29) $\cot \frac{19\pi}{6} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

Unit 5 Pg 17 HW

(5) 
 $\sin \theta = \frac{4}{5}$ $\csc \theta = \frac{5}{4}$
 $\cos \theta = \frac{3}{5}$ $\sec \theta = \frac{5}{3}$
 $\tan \theta = \frac{4}{3}$ $\cot \theta = \frac{3}{4}$

(6) 
 $\sin \theta = 5$ $\csc \theta = \frac{1}{5}$
 $\cos \theta = 0$ $\sec \theta = \text{Und}$
 $\tan \theta = \text{Und}$ $\cot \theta = 0$

(7) 
 $\sin \theta = -\frac{2}{\sqrt{29}}$ $\csc \theta = \frac{\sqrt{29}}{-2}$
 $\cos \theta = \frac{5}{\sqrt{29}}$ $\sec \theta = \frac{\sqrt{29}}{5}$
 $\tan \theta = -\frac{2}{5}$ $\cot \theta = \frac{5}{-2}$

(8) $(0, \frac{\pi}{2})$

- (a) $\sin t = +$
- (b) $\cos t = +$
- (c) $\tan t = +$

(9) $(\frac{\pi}{2}, \pi)$

- (a) $\sin t = +$
- (b) $\cos t = -$
- (c) $\tan t = -$

(10) $(\pi, \frac{3\pi}{2})$

- (a) $\sin t = -$
- (b) $\cos t = -$
- (c) $\tan t = +$

(11) $(\frac{3\pi}{2}, 2\pi)$

- (a) $\sin t = -$
- (b) $\cos t = +$
- (c) $\tan t = -$

(12) $\cos 143 = -$

(16) $\theta = 45^\circ$ A

(18) $\cos 120 = -\frac{1}{2}$

(13) $\tan 192 = +$

(17) $\theta = \frac{7\pi}{6}$ A

(19) $\tan 300 = \frac{-\sqrt{3}}{\frac{1}{2}} = -\sqrt{3}$

(14) $\cos \frac{7\pi}{8} = -$

(20) $\sec \frac{\pi}{3} = \frac{1}{\cos \frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

(15) $\tan \frac{4\pi}{5} = -$

(21) $\csc \frac{3\pi}{4} = \frac{1}{\sin \frac{3\pi}{4}} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$ or $\sqrt{2}$

(22) $\sin \frac{13\pi}{6} = \frac{1}{2}$

(23) $\cos \frac{7\pi}{3} = \frac{1}{2}$

(24) $\tan(-\frac{15\pi}{4}) = 1$

(25) $\cot \frac{13\pi}{4} = 1$

(26) $\cos \frac{23\pi}{6} = \frac{\sqrt{3}}{2}$

(27) $\cos \frac{17\pi}{4} = \frac{\sqrt{2}}{2}$

(28) $\sin \frac{11\pi}{3} = -\frac{\sqrt{3}}{2}$

(29) $\cot \frac{19\pi}{6} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{2}} = -\frac{1}{\sqrt{2}} \cdot \frac{2}{2} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$