Precalculus *Unit 5* *Notes—Angles and their Measures*

Angles are measured a couple of different ways. The first unit of measurement is a degree in which $\underline{360^\circ}$ (degrees) is equal to one revolution. Most likely the reason why we use 360 is from the Babylonians, whose year is based on 360 days. Another unit of measurement for angles is radians. In radians, $\underline{2\pi}$ is equal to one revolution.

So a conversion between radians and degrees $360° = 2\pi$ or $180° = \pi$

When converting from degrees to radians:	When converting from radians to degrees:
Multiply your degrees by $\frac{\pi}{180^\circ}$	Multiply your radians by $\frac{180}{\pi}$

Ex 1) a) Convert 120° to radians

$120^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{3}$ radians

b) Convert $\frac{4\pi}{3}$ into degrees.

$\frac{4\cancel{\pi}}{\cancel{3}} \cdot \frac{\overset{60}{\cancel{180}}}{\cancel{\pi}} = 240^\circ$

c) How many radians are in 90°?

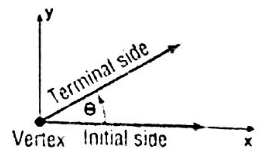
$90^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{2}$ radians

d) How many degrees are in $\frac{\pi}{3}$ radians?

$\frac{\cancel{\pi}}{3} \cdot \frac{180}{\cancel{\pi}} = 60^\circ$

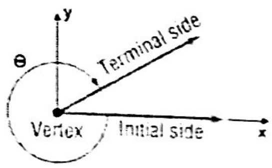
We will use $\underline{\Theta}$ (theta) to represent an angle's measurement. In the figure below it describes how you know if an angle is positive or negative.

The $\underline{\text{vertex}}$ of the angle is at the origin of a rectangular coordinate system. The positive x-axis is always where an angle is measured from, and this is called the $\underline{\text{initial side}}$. An angle drawn this way is said to be in **standard form.**



Counterclockwise rotation

Positive angle



Clockwise rotation

Negative angle

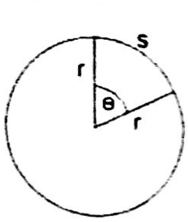
An angle that goes counterclockwise is always positive, and clockwise angles are negative.

<u>Arc Length Formula</u>(*Radian Measure*)

The length of the arc between the two lines (intercepted arc) shown with θ. The equation:

$$s = r\theta$$

where S is the arc length, r is the radius, and θ MUST be measured in RADIANS! The θ is also called the central angle.



Ex 2) a) Find the length of an arc intercepted by a central angle of ½ radian in a circle with radius 5in.

$S = r\Theta$

$S = 5(.5)$

$S = 2.5$ in

b) Find the radian measure of a central angle intercepting an arc length 18 meters in a circle of radius 3 meters.

$S = 18$m

$r = 3$m

$S = r\Theta$

$\frac{18}{3} = \frac{3\Theta}{3}$

$\Theta = 6$ radians

c) Find the length of an arc intercepted by a central angle of $\frac{120}{\pi}$ degrees in a circle with a radius of 7ft.

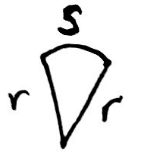
$S = r\theta$

$S = 7 \cdot \frac{2}{3}$

$S = \frac{14}{3}$ ft.

$\frac{120}{\cancel{\pi}} \cdot \frac{\cancel{\pi}}{180} = \frac{2}{3}$ radians

d) Find the perimeter of a 60° slice of a large (7 in. radius) pizza.



$60° \cdot \frac{\pi}{180°} = \frac{\pi}{3}$

$S = r\theta$

$S = 7 \cdot \frac{\pi}{3}$

$S = \frac{7\pi}{3}$

$P = r + r + S$

$P = 7 + 7 + \frac{7\pi}{3}$

$\boxed{P = 14 + \frac{7\pi}{3} \text{ in}}$

e) The running lanes at Emery Sears track at Bluffton College are 1m wide. The inner radius of lane 1 is 33 meters. If the inner radius of lane 2 is 34 meters, how much longer is lane 2 than lane 1?

Looking at the 1st turn in the track as a semi-circle

$L_1: C = 2\pi r$

$C = 2\pi \cdot 33$

$C = 66\pi$

$L_2: C = 2\pi r$

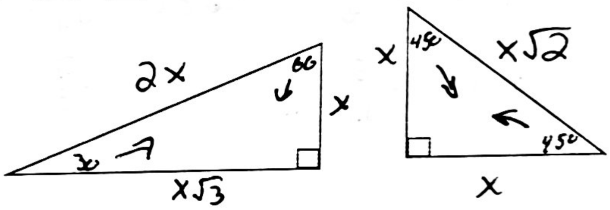
$C = 2\pi(34)$

$C = 68\pi$

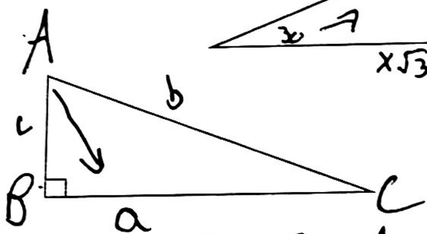
$68\pi - 66\pi = \boxed{2\pi}$

The Pythagorean Theorem: ONLY works for right triangles!!!

The 2 Special Triangles: 30°- 60°- 90° and 45°- 45°- 90°



The Nomenclature of the sides:



The 6 Trig functions: (S O H - C A H - T O A)

Sine(θ) = sinθ = $\frac{opp}{hyp}$ Cosine(θ) = cosθ = $\frac{Adj}{hyp}$ Tangent(θ) = tanθ = $\frac{opp}{adj}$

Cosecant(θ) = cscθ = $\frac{hyp}{opp}$ Secant(θ) = secθ = $\frac{hyp}{adj}$ Cotangent(θ) = cotθ = $\frac{adj}{opp}$

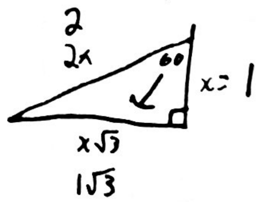
Ex3) Find the value of all 6 trig functions for 45°.

$\sin\theta = \frac{1}{\sqrt{2}}$ $\csc\theta = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\sec\theta = \sqrt{2}$

$\tan\theta = \frac{1}{1}$ $\cot\theta = 1$

Ex4) Find the value of all 6 trig functions for $\frac{\pi}{3}$ radians. 60°



$\sin\theta = \frac{\sqrt{3}}{2}$ $\csc\theta = \frac{2}{\sqrt{3}}$

$\cos\theta = \frac{1}{2}$ $\sec\theta = \frac{2}{1}$

$\tan\theta = \frac{\sqrt{3}}{1}$ $\cot\theta = \frac{1}{\sqrt{3}}$

Ex5) Find the value of all 6 trig functions for $\frac{\pi}{6}$ radians.

Ex6) Find the value of all 6 trig functions for θ.

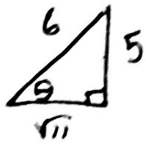
$\sin\theta = \frac{\sqrt{7}}{4}$

$\cos\theta = \frac{3}{4}$

$\tan\theta = \frac{\sqrt{7}}{3}$

Ex7) Let θ be an acute angle such that $\sin\theta = \frac{5}{6}$.

Evaluate the other trig functions of θ.



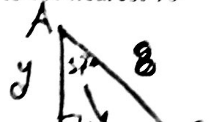
$\sin\theta = \frac{5}{6}$ $\csc\theta = \frac{6}{5}$

$\cos\theta = \frac{\sqrt{11}}{6}$ $\sec\theta = \frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11}$

$\tan\theta = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$ $\cot\theta = \frac{\sqrt{11}}{5}$

Ex8) Find the other five trig functions of angle θ given that $\cos\theta = \frac{3}{7}$.

Ex9) ΔABC is a right triangle with hypotenuse AB 8 in, and $\angle A = 37°$. Draw a diagram, label it, and solve the triangle. (find the measures of all sides & angles). Write answers in both EXACT form & ROUNDED to the nearest 10th



$\sin 37 = \frac{x}{8}$ $\cos 37 = \frac{y}{8}$

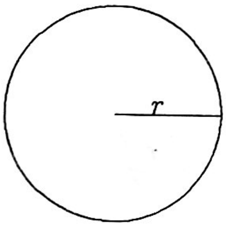
$x = 8\sin 37 \approx 4.8$ $y = 8\cos 37 \approx 6.4$

$B = 53°$

$$\boxed{\begin{array}{ll} A = 37° & a \approx 4.8 \\ B = 53° & b \approx 6.4 \\ C = 90° & c = 8 \end{array}}$$

Think Radian Worksheet

1. What is the circumfrence?



2πr

2. If you measured the circumfrence in terms of the circle's own radius (r), how many are there going once around the circle (360°) 2π

3. How many radians are there in 360°?

2π

4. How many radians are there in a straight angle (180°)?

π

5. How many radians in a right angle?

π/2

6. How many radians is each angle of an equalateral triangle?

π/3

7. The minute hand of a clock travels how many radians in 15 minutes?

π/2

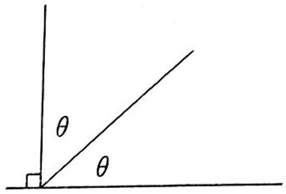
8. The minute hand of a clock travels how many radians in 10 minutes?

π/3

9. The minute hand of a clock travels how many radians in 5 minutes?

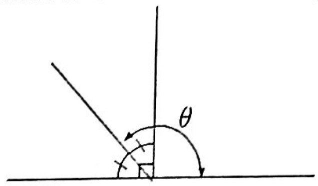
π/6

10. The measure of θ (in radians):



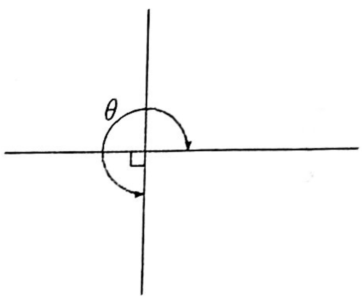
π/4

11. What is the measure of θ (in radians)?



3π/4

12. What is the measure of θ (in radians)?



3π/2