Pre-Calculus

Name: ______________________________

Notes (2.1)---Review Linear & Quadratic Polynomial Functions

Read the definition of a polynomial:

Sometimes it is actually easier to look for what makes a function NOT a polynomial functions rather than the other way around.

DEFINITION

Let n be a nonnegative integer and let $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$ be real numbers with $a_n \neq 0$. The function given by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

is a **polynomial function of degree n**. The **leading coefficient** is a_n.

The zero function $f(x) = 0$ is a polynomial function. It has no degree and no leading coefficient.

List some things to look for that prove a function is NOT a polynomial: inverse functions & exponential functions ~~square~~ rational exponent

Ex1) Determine whether each of the following is a polynomial function, if it is not state why it isn't:

a) $f(x) = 5x^{-1}$ No $\frac{5}{x}$

b) $g(x) = 4x^2 + ex - 10$ yes

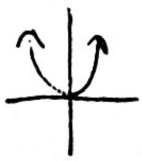
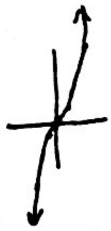
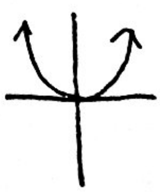
c) $h(x) = -3x^{\pi} + 4x^3 + 11x$ yes

NOW YOU TRY☺

d) $j(x) = 2x^{1/2} + 10x + 6$

e) $k(x) = \frac{1}{4}x^4 - 9x^3 + 7$ yes

f) $f(x) = -x^4 + 5x^3 - 8ix^2 + 7$ No

	Name		Example	Sketch
Polynomial of Degree = 0 → Called a "	Constant	" function → $f(x) =$	4	
Polynomial of Degree = 1 → Called a "	linear	" function → $f(x) =$	2x	
Polynomial of Degree = 2 → Called a "	Quadratic	" function → $f(x) =$	x^2	
Polynomial of Degree = 3 → Called a "	Cubic	" function → $f(x) =$	x^3	
Polynomial of Degree = 4 → Called a "	Quartic	" function → $f(x) =$	x^4	

****There are two polynomial functions you are expected to be very familiar with… Linear & Quadratic****

LINEAR

Slope-Intercept Form: $y = mx + b$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Slope Equation: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Standard Form: $ax + by = c$

QUADRATIC

Vertex Form: $a(x-h)^2 + k = y$

Standard Form: $y = ax^2 + bx + c$

Finding the Vertex: $\left(\frac{-b}{2a}, y\right)$

Completing the Square:

Ways to solve quadratics: graphing, factoring, completing the square, quadratic formula

Ex.2) Write a linear function that satisfies all of the following conditions.

a) through (0, 3) and (−4, −1)

$m = \frac{3 - -1}{0 - -4} = \frac{4}{4} = 1 \quad y = 1x + 3$

b) through: (0, 2) and (1, −3)

$m = \frac{2 - -3}{0 - 1} = \frac{5}{-1} = -5$

$y = -5x + 2$

c) f(−4)= 2 and f(0)= −5 $\quad y = -\frac{7}{4}x - 5$

$(-4, 2)\ (0, -5)$

$m = \frac{2 - -5}{-4 - 0} = \frac{7}{-4}$

d) f(4)= −2 and f(−4)= −4 $\quad y - y_1 = m(x - x_1)$

$(4, -2)\ (-4, -4)$

$m = \frac{-2 + 4}{4 + 4} = \frac{2}{8} = \frac{1}{4}$

$y - -4 = \frac{1}{4}(x - -4)$

Ex.3) Write an equation for the quadratic function with the given vertex and point.

a) Vertex (0, 0) passing through (−2, 8)

$y = ax^2 + \sout{bx + c}$

$8 = a(-2)^2$

$a = 2$

$y = 2x^2$

b) Vertex (2, 0) passing through (1, 3).

$y = a(x-h)^2 + k$

$3 = a(1-2)^2 + 0$

$a = 3$

$y = 3(x-2)^2$

c)) Vertex (−3, 0) passing through (−5, −4).

$y = a(x-h)^2 + k$

$-4 = a(-5 - -3)^2 + 0$

$-4 = a(4)$

$a = -1$

$y = -(x+3)^2$

d) Vertex (−3, 4) passing through (0, 0)

$y = a(x-h)^2 + k$

$0 = a(0 - -3)^2 + 4$

$-4 = 9a$

$a = -4/9$

$y = -4/9(x+3)^2 + 4$

Ex.4) Use completing the square to write the following equations in vertex form.

a) $y = x^2 + 6x - 11$

$y + 11 + 9 = x^2 + 6x + 9$

$y + 20 = (x+3)^2$

$y = (x+3)^2 - 20$

b) $y = 2x^2 - 12x + 1$

$-1 \quad -1$

$\frac{y-1}{2} = \frac{2x^2 - 12x}{2}$

$\frac{y-1}{2} + 9 = x^2 - 6x + 9$

$\frac{y-1}{2} + 9 = (x-3)^2$

$-9 \quad -9$

$\frac{y-1}{2} = (x-3)^2 - 9$

$y - 1 = 2(x-3)^2 - 18$

$+1 \quad +1$

$y = 2(x-3)^2 - 17$

c) $y = -x^2 - 3x - 5$

$+5 \quad +5$

$\frac{y+5}{-1} = \frac{-x^2 - 3x}{-1}$

$\frac{y+5}{-1} + \frac{9}{4} = x^2 + 3x + \frac{9}{4}$

$\frac{y+5}{-1} + \frac{9}{4} = \left(x + \frac{3}{2}\right)^2$

$\frac{y+5}{-1} = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$

$y + 5 = -\left(x + \frac{3}{2}\right)^2 + \frac{9}{4}$

$-\frac{20}{4} \quad -\frac{20}{4}$

$y = -\left(x + \frac{3}{2}\right)^2 - \frac{11}{4}$

d) $y = \frac{1}{3}x^2 - 4x - 1$

$3y = x^2 - 12x - 3$

$+3 \quad +3$

$3y + 3 + 36 = x^2 - 12x + 36$

$3y + 39 = (x-6)^2$

$-39 \quad -39$

$\frac{1}{3}\left(3y = (x-6)^2 - 39\right)$

$y = \frac{1}{3}(x-6)^2 - 13$

When we are asked to solve a quadratic equation, we are really being asked to find the roots.

Ex.5) Use factoring to solve the following quadratic functions.

a) $10x^2 + 13x = 3$

$10x^2 + 13x - 3 = 0$

$(5x - 1)(2x + 3) = 0$

$x = -\frac{3}{2}$ $x = \frac{1}{5}$

b) $5x^2 - 45 = 0$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = 3$ $x = -3$

c) $8x^2 - 2x - 18 = -15$

$8x^2 - 2x - 3 = 0$

$(4x - 3)(2x + 1) = 0$

$x = \frac{3}{4}$ $x = -\frac{1}{2}$

d) $6x^2 + 3x - 3 = 0$

$2x^2 + x - 1 = 0$

$(2x - 1)(x + 1) = 0$

$x = \frac{1}{2}$ $x = -1$

Ex.6) Use the square root method to solve the following quadratic functions.

a) $\sqrt{(5x-1)^2} = \sqrt{12}$

$5x - 1 = \pm 2\sqrt{3}$

$+1 \quad +1$

$\frac{5x}{5} = \frac{1 \pm 2\sqrt{3}}{5}$

$x = \frac{1 \pm 2\sqrt{3}}{5}$

b) $\sqrt{(x+3)^2} = \sqrt{18}$

$x + 3 = \pm 3\sqrt{2}$

$-3 \quad -3$

$x = -3 \pm 3\sqrt{2}$

c) $x^2 + 10x + 25 = 121$

$\sqrt{(x+5)^2} = \sqrt{121}$

$x + 5 = \pm 11$

$-5 \quad -5$

$x = -5 \pm 11$

$x = 6$
$x = -16$

d) $x^2 + 6x + 9 = 8$

$\sqrt{(x+3)^2} = \sqrt{8}$

$x + 3 = \pm 2\sqrt{2}$

$-3 \quad -3$

$x = -3 \pm 2\sqrt{2}$

Ex.7) Use completing the square to solve the following quadratic functions.

a) $x^2 + 2x - 14 = 0$

$x^2 + 2x = 14$

$+1 \quad +1$

$(x+1)^2 = 15$

$x + 1 = \pm\sqrt{15}$

$x = -1 \pm \sqrt{15}$

b) $2x^2 - 8x - 13 = 7$

$+13 \quad +13$

$\frac{2x^2 - 8x}{2} = \frac{20}{2}$

$x^2 - 4x = 10$

$+4 \quad +4$

$\sqrt{(x-2)^2} = \sqrt{14}$

$x = 2 \pm \sqrt{14}$

c) $-x^2 - 2x + 5 = 0$

$-x^2 - 2x = -5$

$x^2 + 2x = 5$

$+1 \quad +1$

$\sqrt{(x+1)^2} = \sqrt{6}$

$x + 1 = \pm\sqrt{6}$

$x = -1 \pm \sqrt{6}$

d) $-2x^2 + 6x + 9 = 0$

$\frac{-2x^2 + 6x}{-2} = \frac{-9}{-2}$

$x^2 - 3x = \frac{9}{4}$

$+\frac{9}{4} \quad +\frac{9}{4}$

$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{\frac{18}{4}}$

$x - \frac{3}{2} = \pm\frac{3\sqrt{2}}{2}$

$+\frac{3}{2} \quad +\frac{3}{2}$

$x = \frac{3 \pm 3\sqrt{2}}{2}$

Graphs of Power Functions and Monomials

This assignment is about the graphs of two particular kinds of polynomials:

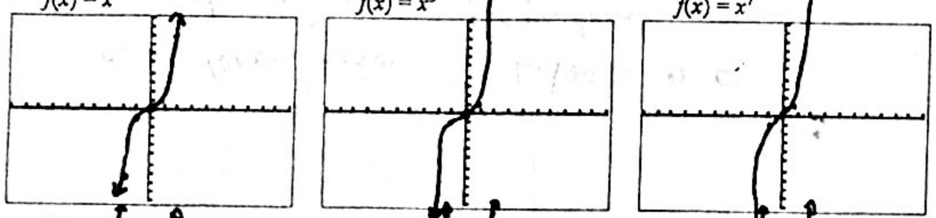
Power functions: A *power function* is a polynomial of the form $f(x) = x^n$.
We've previously studied x^2, but what about other powers: x^3, x^4, x^5, etc.
What can we find out about the graph of x^n in general?

Monomials: A *monomial* is a one-term polynomial, having the form $f(x) = a\,x^n$.
That is, a monomial is just power function multiplied by a number a.
For example, $f(x) = 5x^3$ and $f(x) = -2x^4$ are power functions.
What do the graphs of these functions look like?

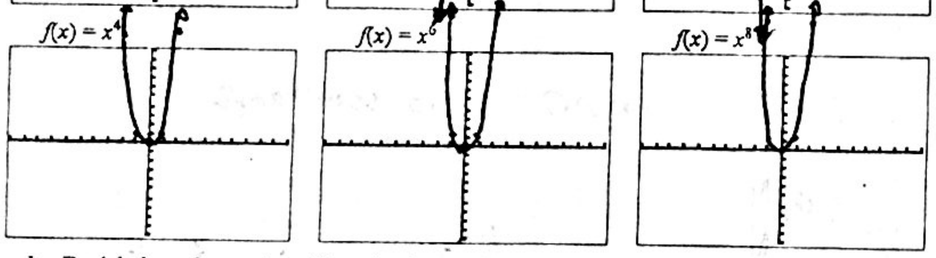
Investigation: graphs of power functions

1. a. Graph the functions shown below on your calculator, which are all of the form $f(x) = x^n$.

$f(x) = x^3$ $f(x) = x^5$ $f(x) = x^7$



$f(x) = x^4$ $f(x) = x^6$ $f(x) = x^8$

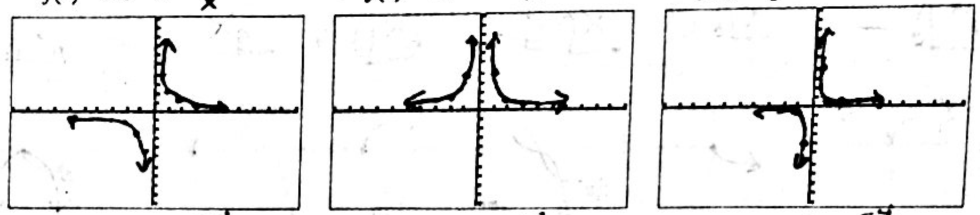


b. Explain how the n value affects the shape of the graph.

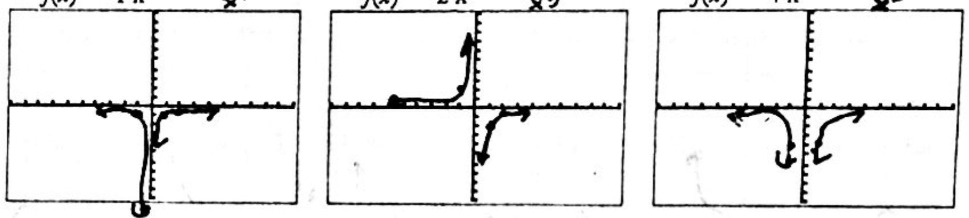
Investigations: graphs of monomials

2. a. Graph the functions shown below on your calculator, which are all of the form $f(x) = ax^n$ where n is negative.

$f(x) = 2x^{-1} = \frac{2}{x}$ $f(x) = 4x^{-2} = \frac{4}{x^2}$ $f(x) = \frac{1}{2}x^{-3} = \frac{1}{2} \cdot \frac{1}{x^3}$



$f(x) = -1\,x^{-4} = \frac{-1}{x^4}$ $f(x) = -2\,x^{-5} = \frac{-2}{x^5}$ $f(x) = -4\,x^{-6} = \frac{-4}{x^6}$



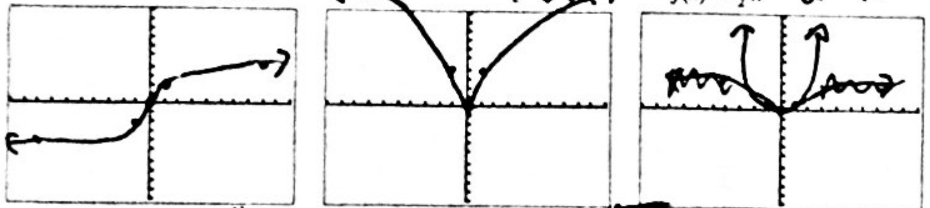
b. Explain how the a value affects the shape of the graph.

c. Explain how the n value affects the shape of the graph.

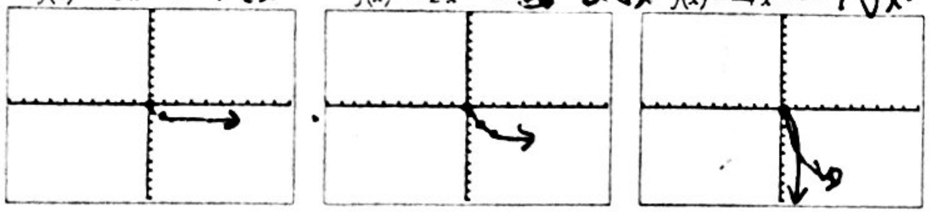
odd → Q1 + Q3
even → Q1 + Q2

3. **a.** Graph the functions shown below on your calculator, which are all of the form $f(x) = ax^n$ where n is not an integer.

$f(x) = 2x^{1/3}$ = $2\sqrt[3]{x}$ $f(x) = 4x^{2/3}$ = $4\sqrt[3]{x^3}$ $f(x) = \frac{1}{2}x^{4/3}$ $\frac{1}{2}\sqrt[3]{x^4}$



$f(x) = -1\,x^{1/4}$ 2 $-1\sqrt[4]{x}$ $f(x) = -2\,x^{3/4}$ = [illegible] $-2\sqrt[4]{x^3}$ $f(x) = -4\,x^{7/4}$ $-4\sqrt[4]{x^7}$



b. Explain how the a value affects the shape of the graph.

steepness of the curve

c. Explain how the n value affects the shape of the graph.

odd denominator 2 halves or 2Q[illegible]

even denominator 1 Quadrant

Summary: End Behavior

The term *end behavior* refers to whether each end of a graph goes up or down. For example, the end behavior of $f(x) = x^3$ is: down on the left, up on the right. Here is what can be seen about end behavior in the graphs you made in problems 1–3.

Right End Behavior: The right end behavior of $f(x) = ax^n$ depends on whether *a* is + or −.

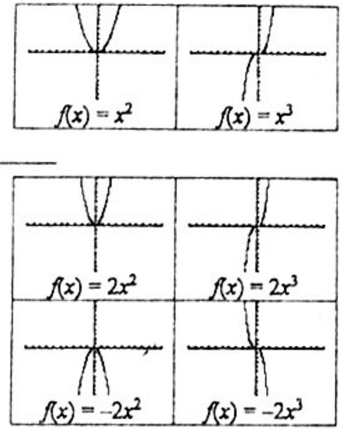
When *a* is +, the right end behavior is up

When *a* is −, the right end behavior is down

Left End Behavior: The left end behavior of $f(x) = ax^n$ depends on whether *n* is even or odd.

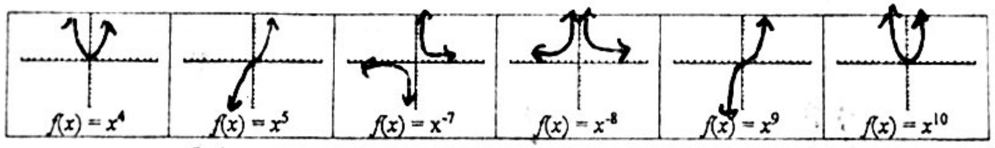
When *n* is even, the left end behavior is same direction as REB

When *n* is odd, the left end behavior is opposite direction of REB

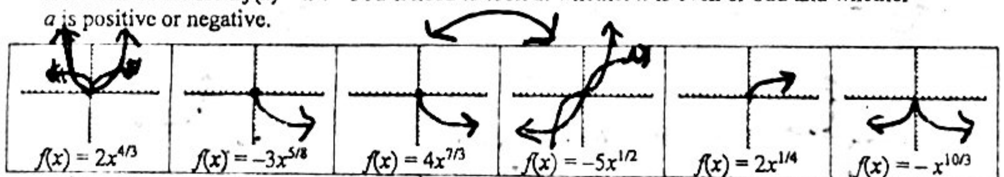


You try it

4. Without using a calculator and without looking back at earlier pages, make a rough sketch of the graph for each of the following functions of the form $f(x) = x^n$. Follow the rules about end behavior. You'll need to look at whether *n* is even or odd.



5. Again, without using a calculator, make a rough sketch of the graph for each of the following functions of the form $f(x) = x^n$. You'll need to look at whether *n* is even or odd and whether *a* is positive or negative.



$2\sqrt[3]{x^4}$ $-3\sqrt[8]{x^5}$ $4\sqrt[3]{x^7}$ $-5\sqrt{x}$ $2\sqrt[4]{x}$ $-\sqrt[3]{x^{10}}$

Pre-Calculus

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Notes (A.2): Factoring Polynomial Functions

	Example #1	Example #2	Example #3	Example #4
	$x^6 - 16x^2$	$-7y^4 - 56y$	$8x^2y - 20xy - 12y$	$3x^3 + 15x^2 - 12x - 60$
Factor out the GCF (if there is one) →	$x^2(x^4-16)$	$-7y(y^3+8)$	$4y(2x^2-5x-3)$	$3(x^3+5x^2-4x-20)$
	2 terms	2 terms	3 terms	4 terms

Identify the method of factoring by the number of terms. → **Binomial** (2 terms) means either or | **Trinomial** (3 terms) means either or | **Polynomial** (4+ terms)

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- The difference of squares $(a^2 - b^2) = (a + b)(a - b)$
  - $x^2 - 16$
  - $(x+4)(x-4)$
- The sum or difference of cubes $(a^3 \pm b^3) = (a \pm b)(a^2 \mp ab + a^2)$
  - $x^3 + 8$
  - $(x+2)(x^2-2x+4)$
- Slide & Divide
  - $2x^2 - 7x - 9$
  - $x^2 - 7x - 18$
  - $(x - \frac{9}{2})(x + \frac{2}{2})$
  - $(2x-9)(x+1)$
- Split the Middle Term
  - $2x^2 - 7x - 9$
  - $2x^2 + 2x - 9x - 9$
  - $2x(x+1) - 9(x+1)$
  - $(2x-9)(x+1)$
- Grouping

Always check to see if there is more factoring to do… ☺ →

NOW YOU TRY ☺

1) $-2x^3 + 2x$

$-2x(x^2-1)$

$-2x(x-1)(x+1)$

2) $54x^3 - 128$

$2(27x^3-64)$

$2(3x-4)(9x^2+12x+16)$

3) $-36x^3y + 15x^2y + 6xy$

$-3xy(12x^2-5x-2)$

$-3xy(4x+1)(3x-2)$

$-3xy(4x+1)(3x-2)$

4) $60x^3 + 40x^2 - 135x - 90$

$5(12x^3 + 8x^2 - 27x - 18)$

$5(4x^2(3x+2) - 9(3x+2))$

$5(4x^2-9)(3x+2)$

$5(2x-3)(2x+3)(3x+2)$

5) $x^4 - 29x^2 + 100$

$(x^2-25)(x^2-4)$

$(x+5)(x-5)(x+2)(x-2)$

Pre-Calculus Name: ____________________

Notes (2.3) – Polynomial Functions

Graphing Polynomial Functions

Investigation 1 - End Behavior:

1. Using a graphing calculator, graph the following functions:

a. $y = 3x^4 - 7x^3 + x^2 + 9$
b. $y = -1/2x^6 - 4x^5 + 2x^3 - 11x + 5$
c. $y = 2x^3 + 5x2 - 3x + 1$
d. $y = -3x^5 + 7x^3 - 5$

2. What affects the right end behavior?

+ & −

3. What affects the left end behavior?

even & odd degree & ±

****

*******Graphing Polynomial Functions*******

Not only are graphs of polynomials unbroken without jumps or holes, but they are *smooth*, unbroken lines or curves, with no sharp corners or cusps.

****

THEOREM ----A polynomial function of degree __n__ has at most __n-1__ local extrema and at most __n__ zeros.

End Behavior of Polynomial Functions

In order to determine the end behavior of a polynomial function you need only 2 pieces of information:

1[st]: You must know the __degree__ of the polynomial. If the __degree__ is __even__ the LEFT END BEHAVIOR (L.E.B) & the RIGHT END BEHAIVOR (R.E.B) will be __the same__. If the __degree__ is __odd__ then the L.E.B. and the R.E.B. will be __opposite__.

2[nd]: You must know the sign of the __Leading__ __coefficient__ (__L__.__C__.) of the polynomial. If the __L__.__C__. is **POSITIVE** then the R.E.B. will be: $\lim_{x\to\infty} f(x) =$ __∞__. However, if the __L__.__C__. is **NEGATIVE** then the R.E.B. will be: $\lim_{x\to\infty} f(x) =$ __−∞__.

Zeros of Polynomial Functions

Investigation 2 - Local Extrema and Zeros

Zeros of Polynomial Functions

Recall that finding the real-number zeros of a function f is equivalent to finding the x-intercepts of the graph of $y = f(x)$ or the solutions to the equation $f(x) = 0$.

a) Investigate various third degree polynomial functions using the graphing calculator to see how many extrema and how many zeros the function can have. Sketch a graph below.

b) Investigate various sixth degree polynomial functions using the graphing calculator to see how many extrema and how many zeros the function can have. Sketch a graph below.

➔ In general, if your degree is "n", how many extrema are possible? $n-1$

➔ In general, if your degree is "n", how many zeros are possible? n

EXAMPLE: Finding the Zeros of a Polynomial Function

Find the zeros of $f(x) = x^3 - x^2 - 6x$ and then sketch the graph of the polynomial using your knowledge of intercepts and end behavior (so, not a calculator).

$x(x^2 - x - 6) = 0$ $\quad x = 0 \quad x - 3 = 0 \quad x + 2 = 0$

$x(x-3)(x+2) = 0$ $\quad x = 0 \quad x = 3 \quad x = -2$

****

*****DEFINITION Multiplicity of a Zero of a Polynomial Function*****

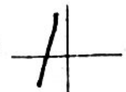
If f is a polynomial function and $(x - c)^m$ is a factor of f, then c is a zero of **multiplicity m** of f.

****

Zeros of Odd and Even Multiplicity

If a polynomial function f has a real zero c of odd multiplicity, then the graph of f crosses the x-axis at $(c, 0)$ and the value of f changes sign at $x = c$. If a polynomial function f has a real zero c of even multiplicity, then the graph of f does not cross the x-axis at $(c, 0)$ and the value of f does not change sign at $x = c$.

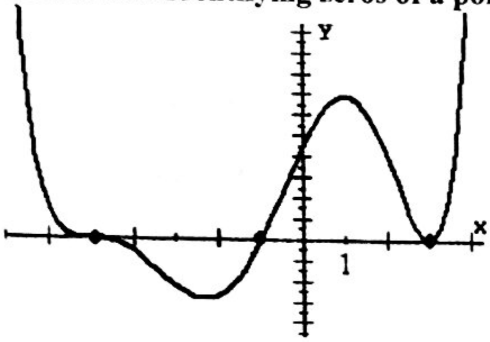
So… If the multiplicity of a zero is 1 it will cross the x-axis in the typical "straight through" manner.



……If the If the multiplicity of a zero is EVEN it will bounce at the x-axis & will NOT cross through.

…if the If the multiplicity of a zero is **GREATER THAN 1 & ODD** it will slide at the x-axis & WILL cross through.

EXAMPLE: Identifying zeros of a polynomial function.



Multiplicity of 1: x = -1

Even Multiplicity: x = 3

Odd Multiplicity > 1: x = -5

Sketching the Graph of a Factored Polynomial

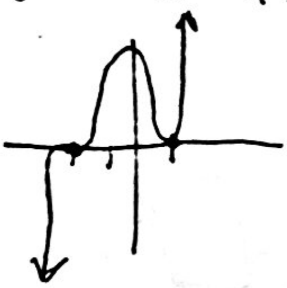
EXAMPLE: Sketching the Graph of a Factored Polynomial

State the degree and list the zeros of the function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of f by hand.

(a) $f(x) = (x+2)^3(x-1)^2$

d: 5

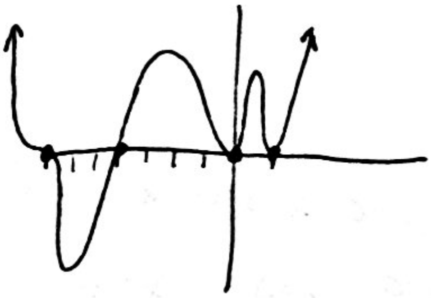
Z: x = -2(3) x = 1(2)



(b) $f(x) = x^2(x+7)^3(x-1)^4(x+4)$

d=10

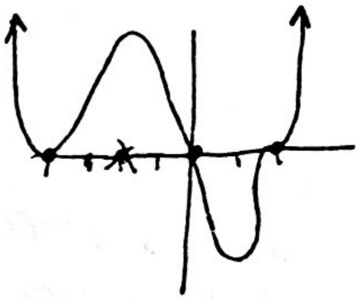
Z: x=0 m2 x=-7 m3 x=1 m4 x=-4



(c) $f(x) = x(x+4)^2(x-2)^3$

d: 6

Z: x=0 x=-4 M2 x=2 m3

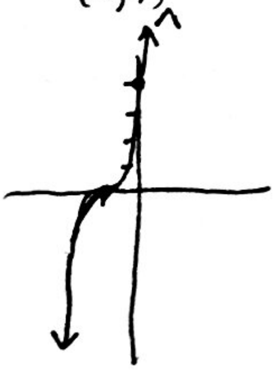


Graphing Transformations of Power Functions

Describe how to transform the graph of an appropriate monomial function $f(x) = ax$ into the graph of the given function. Sketch the transformed graph by hand compute the location of the y-intercept to check on the graph.

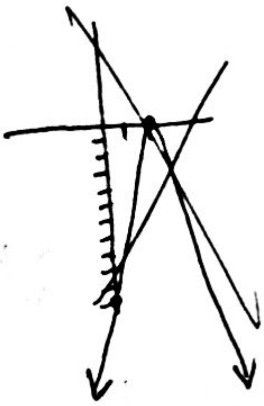
(a) $g(x) = 4(x+1)^3$

(0,4)



(b) $h(x) = -(x-2)^4 + 5$

(0,-11)



(c) $f(x) = -2(x-3)^5 - 1$

(d) $k(x) = (x-3)^{2/3} + 2$

Precalculus

Real Zeros of Polynomials (2.4)

Remember long division?

How about with polynomials?

Ex1) $\underline{1373}$ ______

$$\begin{array}{r} 457 \text{ R}2 \\ 3\overline{)1373} \\ -12\downarrow \\ 17 \\ -15\downarrow \\ 23 \\ 21 \\ \hline 2 \end{array}$$

or $457\frac{2}{3}$

Ex2) $\dfrac{3x^2-7x+2}{x+5}$ ______

$3x-22+\dfrac{112}{x+5}$

$$\begin{array}{r} 3x-22 \\ x+5\overline{)3x^2-7x+2} \\ -(3x^2+15x)\downarrow \\ \hline -22x+2 \\ -(-22x-110) \\ \hline 112 \end{array}$$

STEP#1: Determine what you can multiply the 1st term in **DIVISOR** by to get as close to the first term of the **DIVIDEND** as possible.

STEP#2: **Multiply the whole DIVISOR** by that amount

STEP #3: Subtract

STEP#4: Bring down next term

STEP#5: REPEAT

**** Note: Write your final answer as the *QUOTIENT* + $\dfrac{REMAINDER}{DIVISOR}$

Ex3) $\dfrac{2x^4-x^3-2}{2x^2+x+1}$

$x^2-x+\dfrac{x-2}{2x^2+x+1}$

$$\begin{array}{r} 2x^2+x+1\overline{)2x^4-x^3+0x^2+0x-2} \\ -(2x^4+x^3+x^2)\downarrow \\ \hline -2x^3-x^2+0x \\ -(-2x^3-x^2-x) \\ \hline x-2 \end{array}$$

Place holders are NECESSARY when you are "missing" terms.

QUESTION: So when do you get to use *SYNTHETIC DIVISION*?

→ **ANSWER**: Whenever your **DIVISOR** is linear (aka in the form mx + b)

Ex4) Use BOTH long and synthetic to find the following quotient: $\frac{2x^3-3x^2-5x-12}{x-3}$

LONG DIVISION

$$\begin{array}{r} 2x^2+3x+4 \\ x-3\overline{)2x^3-3x^2-5x-12} \\ -(2x^3-6x^2) \\ \hline 3x^2-5x \\ -(3x^2-9x) \\ \hline 4x-12 \\ -(4x-12) \\ \hline 0 \end{array}$$

SYNTHETIC DIVISION

3	2	-3	-5	-12
		6	9	12
	2	3	4	0

$2x^2+3x+4$

QUESTION: Why do we use long and synthetic division?

→ **ANSWER**: Mainly to find zeros of polynomials that we could not factor using any of the methods that we have learned before.

QUESTION: If the process of long and synthetic division is "embedded" in a problem with a polynomial then how will we know what to divide by (since it will not be so specific)?

→ **ANSWER**: Use the RATIONAL ROOT THEOREM

THEOREM: To identify all POSSIBLE RATIONAL ZEROS (again these are POSSIBLE NOT definitely zeros), we begin by listing all the factors of the constant term, factors of our leading coefficient, and then we create the list of possibilities using by finding ALL COMBINATIONS of these factors using the ones from the constant term as "numerators" and factors from leading coefficient as "denominators." Then we simplify all of these numbers, eliminate repeats, and add "plus or minus" to each of them.

Ex5) List all possible rational zeros of $f(x)=3x^4+2x^3-7x+6$

STEP #1: List factors of constant-----------------→ $\pm1, \pm2, \pm3, \pm6$

STEP #2: List factors of leading coefficient-----→ $\pm1\ \pm3$

STEP #3: Write out all combos using factors of Constant as "numerators" & factors of the Leading Coefficient as "denominators" ---------→ $\pm\frac{1}{1}\ \pm\frac{2}{1}\ \pm\frac{3}{1}\ \pm\frac{6}{1}\ \pm\frac{1}{3}\ \pm\frac{2}{3}\ \pm\frac{3}{3}\ \pm\frac{6}{3}$

STEP #4: Simplify ALL, eliminate repeats & add "plus or minus" to each--------→ $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{3}, \pm\frac{2}{3}$

QUESTION: Why do we make that list again?

→ **ANSWER**: To have some numbers to ___√___ to see if they are ___0's___ of the polynomial.

Ex6) Find the zeros of $f(x)=x^3-3x^2+1$

$\frac{\pm 1}{\pm 1}$

1 | 1 -3 0 1
1 -2 -2
1 -2 -2 -1

-1 | 1 -3 0 1
-1 4 -4
1 -4 4 -3

Ex7) Find all rational zeros of $f(x)=3x^3+4x^2-5x-2$

$\frac{\pm 1 \ \pm 2}{\pm 1 \ \pm 3}$

1 | 3 4 -5 -2
3 7 2
3 7 2 0

$3x^2+7x+2$

$-\frac{1}{3}$ | 3 7 2
-1 -2
$\frac{3}{3}$ $\frac{6}{3}$ 0

1 2

1x + 2

$x=-\frac{1}{3}$ $x=-2$ $x=1$

$3x=-1$ $x+2=0$ $x-1=0$

$3x+1=0$ $x+2=0$ $x-1=0$

$(3x+1)(x+2)(x-1)$

$x=-2$

$x=-\frac{1}{3}$

$x=1$

Ex8) Find all real zeros of $f(x)=2x^4-7x^3-8x^2+14x+8$

$\frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1 \ \pm 2}$

1 | 2 -7 -8 14 8
2 -5 -13 1
2 -5 -13 1 9

-1 | 2 -7 -8 14 8
-2 9 -1 -13
2 -9 1 13 -5

2 | 2 -7 -8 14 8
4 -6 -28 -28
2 -3 -14 -14 -20

-2 | 2 -7 -8 14 8
-4 22 -28 28
2 -11 14 -14 36

4 | 2 -7 -8 14 8
8 4 -16 -8
2 1 -4 -2 0

$2x^3+1x^2-4x-2=0$

$x^2(2x+1)-2(2x+1)=0$

$(x^2-2)(2x+1)=0$

$x=4$ $x=-\frac{1}{2}$ $x=\pm\sqrt{2}$

COMPLEX NUMBERS REVIEW (2.5)

RECALL – The *imaginary number* is represented by the letter i

$i = \sqrt{-1}$

$i^2 = -1$

Now keeping in mind what we are allowed to do with equations. What if we square both sides of that?

So that means that i^2 = -1 which is pretty interesting, because -1 is a REAL number!

RECALL - *Complex numbers*: These are numbers that have 2 parts to them, a real part, and an imaginary part.

The standard form of a complex number is $a \pm bi$. Where a - real # & bi - imaginary.

*********NOTE: complex numbers should ALWAYS be written in their standard form********

OPERATIONS WITH COMPLEX NUMBERS

Essentially, when you are working with complex numbers, just follow all the rules of ALGEBRA you have learned up to this point, treating the i as if it were a variable (like x). Then, when you have finished your operation, and you are simplifying, replace any i^2 with -1 & finish simplifying.

***REMEMBER – you have to write your answer in the standard form, so real terms first then imaginary.

Practicing Operations with Complex Numbers:

Ex1) $(7 - 3i) + (4 + 5i) = 11 + 2i$

Ex2) $(5 + 2i) - (2 - i) = 3 + 3i$

Ex3) $(2 + 1)(4 - i) = 8 - 2i + 4 - i \rightarrow 12 - 3i \rightarrow$ ______

Ex4) Given that $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ find the value of z^2 and z^3

$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$\frac{1}{4} + \frac{i\sqrt{3}}{4} + \frac{i\sqrt{3}}{4} + \frac{3}{4}i^2$

$-\frac{2}{4} + \frac{i\sqrt{3}}{2} = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$

$\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$

$-\frac{1}{4} + \frac{i\sqrt{3}}{4} - \frac{i\sqrt{3}}{4} + \frac{i^2 \cdot 3}{4}$

$-\frac{1}{4} - \frac{3}{4} = -1$

$z^2 = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$

$z^3 = -1$

Ok review is almost over… just ONE more thing…do you remember "***complex conjugates***?"

In short, every complex number has a complex conjugate that is ALMOST identical because both the $a + bi$ & $a - bi$ are the same. However, the sign between them changes.

Ex5) Find the complex conjugate of each of the following:

a) $4 - 2i$ → $4 + 2i$

b) $-17 + 47i$ → $-17 - 47i$

QUESTION: Why we need conjugates?

- ➔ **ANSWER:** We need them for division because all of our complex numbers have to be written in standard form. So when we need to get rid of a complex number in the denominator, we multiply (TOP & BOTTOM) by its complex conjugate, simplify, and write in std form.

Ex6) $\frac{2}{3-i} \cdot \frac{3+i}{3+i} = \frac{6+2i}{9+3i-3i-i^2}$

$\frac{6+2i}{10} = \frac{3+i}{5}$

Ex7) $\frac{5+i}{2-3i} \cdot \frac{2+3i}{2+3i} = \frac{10+15i+2i+3i^2}{4-9i^2}$

$= \frac{7+17i}{13}$

Precalculus

COMPLEX ZEROS & THE FUNDAMENTAL THEOREM OF ALGEBRA (2.6)

THEOREM: Fundamental Theorem of Algebra – A polynomial of degree n >0 has exactly n complex zeros.

NOTE: Complex includes: REAL, IMAGINARY, & COMPLEX*

ALSO → the theorem doesn't use the word "distinct" which means they don't all have to be different. In other words we are allowed to have repeated zeros and it will count each and every time

Ex1) How many complex zeros does each of the following have?

A) $f(x) = x^2 + 5$

2

2 imaginary

B) $f(x) = x^3 - 1$ $\quad (x-1)(x^2+x+1)$

3

1 real 2 imaginary

THEOREM: Polynomial Functions of Odd Degree – ≥ 1 real solution

THEOREM: Linear Factorization Theorem – If a polynomial with degree $n>0$ then it has exactly n linear factors

THEOREM: Complex Conjugate Zeros – if $a+bi$ is a solution, then $a-bi$ is a solution

Ex2) Determine the zeros of the following polynomials...you are going to need the quadratic formula:

a) $f(x) = x^2 + x + 1$

$$x = \frac{-1 \pm \sqrt{1-4(1)(1)}}{2(1)}$$

$$x = \frac{-1 \pm i\sqrt{3}}{2}$$

or

$$x = -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$

b) $g(x) = x^3 - 27$

$(x-3)(x^2+3x+9)$

$x=3$

$$x = \frac{-3 \pm \sqrt{9-4(1)(9)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{-27}}{2}$$

$$x = \frac{-3 \pm 3i\sqrt{3}}{2}$$

or

$$x = -\frac{3}{2} \pm \frac{3i\sqrt{3}}{2}$$

c) $h(x) = x^2 + 7$

$\sqrt{x^2} = \sqrt{-7}$

$x = \pm i\sqrt{7}$

What if we were given one of the zeros, but it just happened to be complex.......could you still do it?

Ex3) Given that $f(x) = 4x^4 + 17x^2 + 14x + 65$ and given that one zero of f(x) is $1 - 2i$, find the remaining zeros of f(x) and write it in its linear factorization.

-sum: $1-2i + 1+2i = 2$

+product: $(1-2i)(1+2i)$
$1 + 2i - 2i - 4i^2 = 5$

$x^2 - 2x + 5$

$$\begin{array}{r} 4x^2 + 8x + 13 \\ x^2-2x+5 \overline{)\, 4x^4 + 0x^3 + 17x^2 + 14x + 65} \\ -(4x^4 - 8x^3 + 20x^2) \\ 8x^3 - 3x^2 + 14x \\ -(8x^3 - 16x^2 + 40x) \\ 13x^2 - 26x + 65 \\ -(13x^2 - 26x + 65) \end{array}$$

$4x^2 + 8x + 13$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{64-4(4)(13)}}{2(4)}$

$x = \frac{-8 \pm \sqrt{144}}{8}$

$x = \frac{-8 \pm 12i}{8}$

$x = \frac{-2 \pm 3i}{2}$

$f(x) = (x^2-2x+5)(4x^2+8x+13)$