RIGID TRANSFORMATION—a transformation that will leave the size and shape of a graph unchanged. This includes horizontal translations, vertical translations, reflections, or any combination of these.

NON-RIGID TRANSFORMATION—a transformation which will generally distort the shape of a graph. This includes horizontal or vertical stretches and shrinks.

Transformation	Function	Description
Horizontal Shift	$f(x + h)$	Shift left h units
	$f(x - h)$	Shift right h units
Vertical Shift	$f(x) + k$	Shift up k units
	$f(x) - k$	Shift down k units
Reflection	$-f(x)$	Reflect across x-axis
	$f(-x)$	Reflect across y-axis
Vertical Stretch/Compress	$a\,f(x),\ a > 1$	Stretch vertically by a factor of a
	$a\,f(x),\ 0 < a < 1$	Compress vertically by a factor of a
Horizontal Stretch/Compress	$f(ax),\ a > 1$	Compress horizontally by a factor of $\frac{1}{a}$
	$f(ax),\ 0 < a < 1$	Stretch horizontally by a factor of $\frac{1}{a}$

***NOTE*: If there is a coefficient to x and a horizontal translation (a "b" and an "h") then the coefficient should be factored out in order to truly see what the horizontal shift is.**

Order of function transformations

- Horizontal shifts
- Horizontal stretch/compression
- Reflection over y-axis
- Vertical stretch/compression
- Reflection over x-axis
- Vertical shifts

Example 1 Describe the transformation that occurs from a parent function.

$f(x)=e^x$

a) $f(x)=x^2+3$ $\uparrow 3$

b) $f(x)=x^2-5$ $\downarrow 5$

c) $f(x)=(x-2)^2$ $\rightarrow 2$

d) $f(x)=(x+4)^2$ $\leftarrow 4$

e) $f(x)=3e^x$ vert. stretch by 3

f) $f(x)=\frac{1}{2}e^x$ vert shrink by $\frac{1}{2}$

g) $f(x)=(3x)^2$ horiz shrink by $\frac{1}{3}$

h) $f(x)=\left(\frac{1}{2}x\right)^2$ horiz stretch by 2

i) $f(x)=-|x|$ reflect over x-axis

j) $f(x)=|-x|$ reflect over y-axis

Example 2 Describe how the graph of $y=|x|$ can be transformed to the graph of the given equation.

a) $y=|x|-4$ $\downarrow 4$

b) $y=|x+2|$ $\leftarrow 2$

c) $y=-|x-6|$ reflect over x-axis, $\rightarrow 6$

d) $y=|-x+2|$ $y=|-(x-2)|$ reflect over y-axis, $\rightarrow 2$

e) $y=-|x+3|-7$ reflect over x-axis, $\leftarrow 3$, $\downarrow 7$

Example 3 Find an equation for the following transformations of the function $f(x)=\sqrt{x}$.

a) $f(x)$ is reflected over the y-axis and translated up 3 units

$f(x)=\sqrt{-x}+3$

b) $f(x)$ is vertically stretched by a factor of 3 and translated 4 units left

$f(x)=3\sqrt{x+4}$

c) $f(x)$ is horizontally shrunk by a factor of $\frac{1}{2}$ and reflected over the x-axis

$f(x)=-\sqrt{2x}$

Example 4 Describe the following transformations that have been applied to one of the 12 parent functions.

a) $f(x) = 0.5\sin(2x-6)+7$

$.5\sin(2(x-3))+7$

vert shrink by $\frac{1}{2}$

horiz shrink by $\frac{1}{2}$

$\rightarrow 3 \quad \uparrow 7$

b) $f(x) = -\ln(-x+4)-2$

$-\ln[-(x-4)]-2$

reflect over x-axis

reflect over y-axis

$\rightarrow 4 \quad \downarrow 2$

c) $f(x) = \dfrac{2}{1+e^x}$

vertical stretch by 2

reflect over y-axis

Example 5 Find an equation for the following transformations of the function $f(x) = e^x$.

a) $f(x)$ is reflected over the x-axis and translated down 2 units

$f(x) = -e^x - 2$

b) $f(x)$ is vertically shrunk by a factor of $\frac{1}{4}$ and translated 6 units right

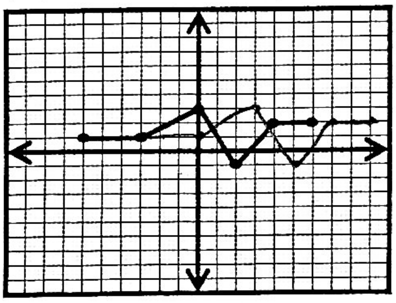
$f(x) = \frac{1}{4}e^{x-6}$

c) $f(x)$ is horizontally stretched by a factor of 7 and shifted up 3 and left 4

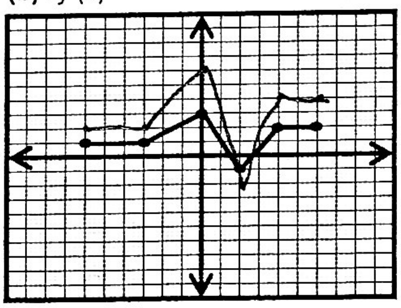
$f(x) = e^{\frac{1}{7}(x+4)} + 3$

Example 6 Given the graph of $f(x)$ in each coordinate plane below, sketch each of the transformations indicated:

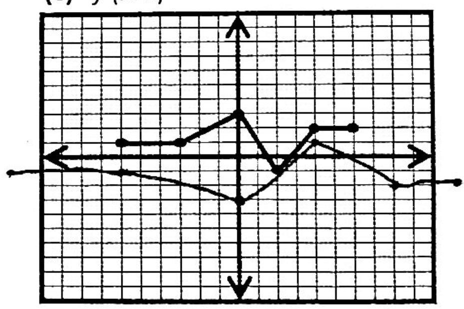
(a) $f(x-3)$



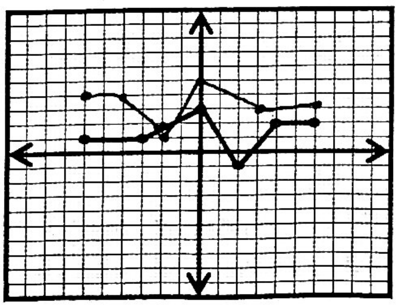
(b) $2f(x)$



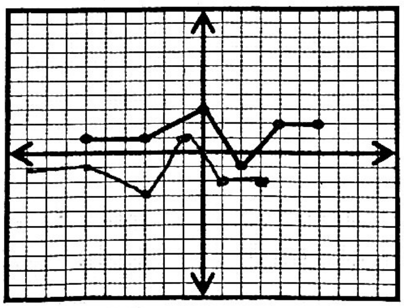
(c) $-f(\frac{1}{2}x)$



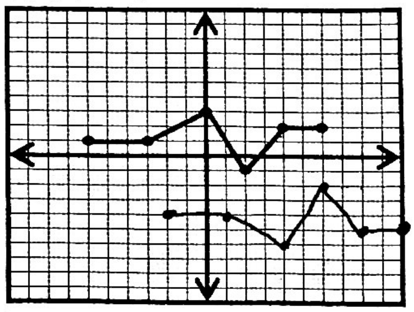
(d) $f(-x)+2$



(e) $-f(x+3)$



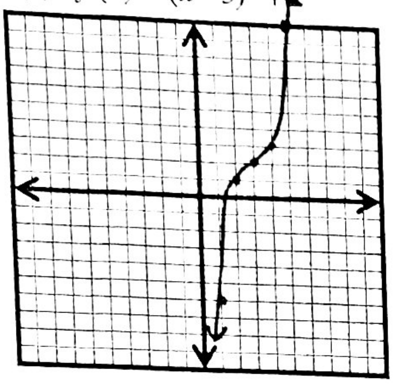
(f) $-f(x-4)-3$



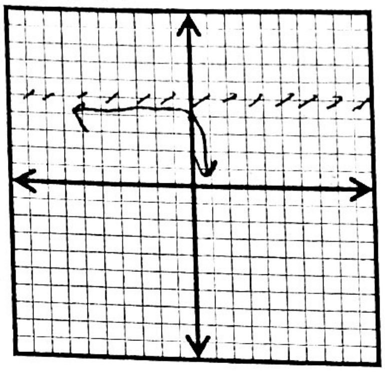
Precalculus *Unit 1* *Homework—Transformations*

Graph each of the following transformations of the 12 basic functions without the use of your calculator (although you may feel free to use your calculator to CHECK and make sure your graphs are correct)

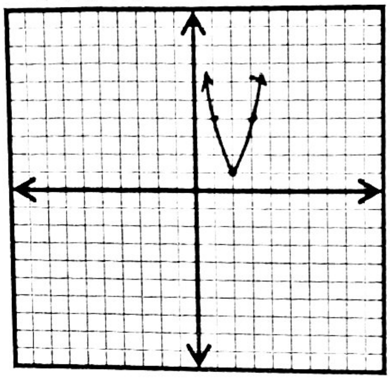
(1) $f(x) = (x-3)^3 + 2$



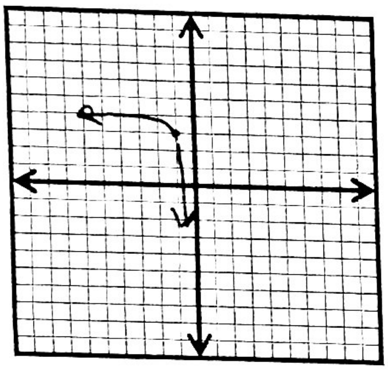
2) $f(x) = -e^{-x} + 5$



(3) $f(x) = 3(x-2)^2 + 1$

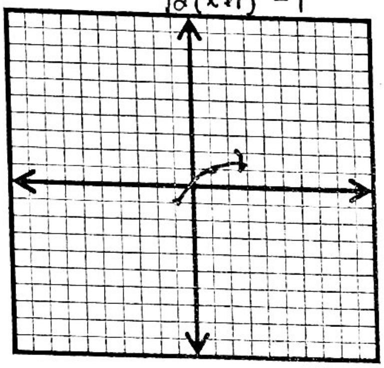


(4) $f(x) = \ln(-x) + 3$

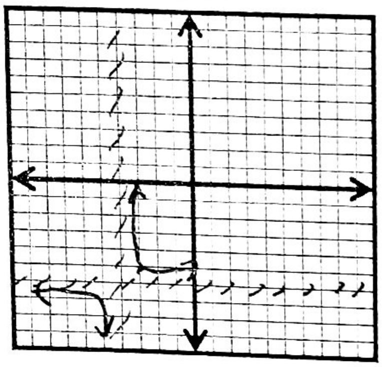


(5) $f(x) = \sqrt{2x+2} - 1$

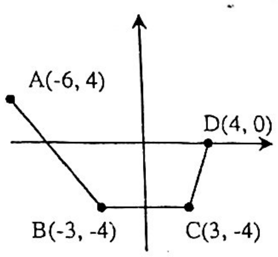
$\boxed{2}(x+1) - 1$



(6) $f(x) = \dfrac{1}{x+4} - 6$



(7) The graph of $f(x)$ is shown below. Find the new coordinates after the following transformations have been applied:



		A	B	C	D
$f(x)$	-----	(–6, 4)	(–3, –4)	(3, –4)	(4, 0)
(a) $-f(x)$	-----	(-6, -4)	(-3, 4)	(3, 4)	(4, 0)
(b) $f(-x)$	-----	(6, 4)	(3, -4)	(-3, -4)	(-4, 0)
(c) $f(x)+3$	-----	(-6, 7)	(-3, -1)	(3, -1)	(4, 3)
(d) $f(x-3)$	-----	(-3, 4)	(0, -4)	(6, -4)	(7, 0)
(e) $2f(x)$	-----	(-6, 8)	(-3, -8)	(3, -8)	(4, 0)
(f) $f(\frac{1}{2}x)$	-----	(-12, 4)	(-6, -4)	(6, -4)	(8, 0)

(8) Describe the transformation(s) of $f(x)$ indicated by each of the following functions:

(a) $f(-x+3)$ $f(-(x-3))$
reflect over y
→ 3

(b) $-3f(x+1)$
reflect over x
vertical stretch by 3
← 1

(c) $f(-3x-12)$ $f(-3(x+4))$
reflect over y-axis
horiz shrink by 3
← 4

(d) $-\frac{1}{2}f(x) - 7$
reflect over x-axis,
vertical shrink by $\frac{1}{2}$
↓ 7

$f(x)=\sqrt{x}$

...ite an equation for each situation described below:

(a) The squaring function is reflected over x-axis and translated up 3 & left 2.

$f(x) =$ $-(x+2)^2+3$

(b) The logistic function vertically stretched by 3 and reflected over the y-axis.

$f(x) =$ $\dfrac{3}{1+e^{x}}$

(c) The square root function is horizontally shrunk by a factor of ¾ & translated right 6 spaces

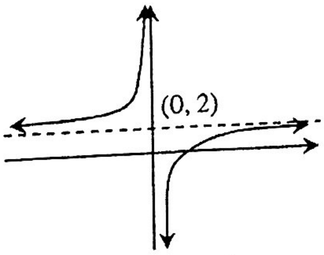
$f(x) =$ $\sqrt{\frac{4}{3}(x-6)}$

(d) The reciprocal function is translated up 9 units and reflected over the y-axis.

$f(x) =$ $\dfrac{1}{-x}+9$

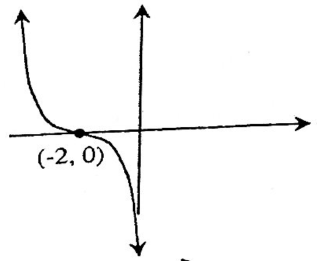
(10) Write an equation for each of the graphs below with the horizontal/vertical stretch/shrink indicated.

(a) Horizontal stretch by a factor of 12



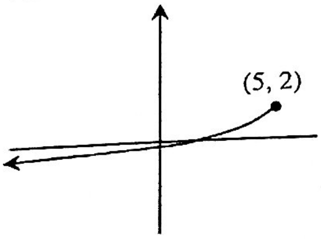
$f(x) =$ $-\dfrac{1}{\frac{1}{12}x}+2$

(b) Vertical shrink by a factor of ¾



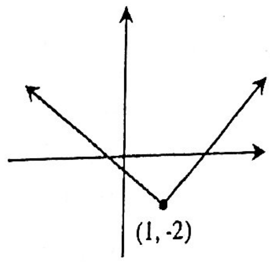
$f(x) =$ $-\frac{3}{4}(x+2)^3$

(c) No horizontal or vertical shrink/stretch



$f(x) =$ $-\sqrt{-(x-5)}+2$

(d) Vertical stretch by a factor of 4



$f(x) =$ $4|x-1|-2$