

Steps to solve a rational equation

- 1) Find the LCD.
- 2) Multiply every term on both sides of the equation by the LCD (objective is to divide out the denominators)
- 3) Solve for the variable.
- 4) Check for extraneous solutions!!! An extraneous solution is a solution to an equation that emerges from the process of solving the problem but is not a valid solution to the problem.

Example 1 Solve.

$$x + \frac{3}{x} = 4$$

check

$$3 + \frac{3}{3} = 4$$

$$1 + \frac{3}{1} = 4$$

STEP #1: Identify the LCD.

$$(x) \left(x + \frac{3}{x} \right) = (4)(x)$$

STEP #2: Multiply both sides by LCD.

(remember to DISTRIBUTE!!)

$$x^2 + 3 = 4x$$

-4x -4x

STEP #3: Solve.

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

STEP #4: Check your answers!!!!

$x = 3 \quad x = 1$

Example 2 Solve.

A. $\left(x + \frac{4}{x-4} = 0 \right) \cdot (x-4)$

$$x(x-4) + \frac{4}{x-4} \cdot (x-4) = 0(x-4)$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)(x-2) = 0$$

$x = 2$

 $x = 2$

Check

$$2 + \frac{4}{2-4} = 0$$

B. $\left(\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2-4x+3} \right) \cdot (x-3)(x-1)$

$$2x(x-3) + 1(x-1) = 2$$

$$2x^2 - 6x + 1x - 1 - 2 = 0$$

$$2x^2 - 5x - 3 = 0$$

$$(2x+1)(x-3) = 0$$

$x = -\frac{1}{2}$

 ~~$x = 3$~~

$$C. \left(\frac{2x+2}{2x+5} - \frac{x-4}{3x-1} = \frac{5x^2+18}{(2x+5)(3x-1)} \right)$$

$$① (2x+2)(3x-1) - (x-4)(2x+5) = 5x^2+18$$

$$② 6x^2 - 2x + 6x - 2 - (2x^2 + 5x - 8x - 20) = 5x^2 + 18$$

$$③ 6x^2 + 4x - 2 - (2x^2 - 3x - 20) = 5x^2 + 18$$

$$④ 4x^2 + 7x + 18 = 5x^2 + 18$$

$$-4x^2 - 7x - 18 - 4x^2 - 18 - 7x$$

$$⑤ 0 = x^2 - 7x$$

$$x(x-7) = 0$$

$$x=0 \quad x=7$$

$$E. \left(\frac{5x}{x-2} = 7 + \frac{10}{x-2} \right) x-2$$

$$5x = 7(x-2) + 10$$

$$5x = 7x - 14 + 10$$

$$-5x - 5x$$

$$0 = 2x - 4$$

$$\cancel{x=2}$$

No Solution

$$D. \left(\frac{t+4}{t} + \frac{-4}{t-4} = \frac{-16}{t^2-4t} \right) t(t-4)$$

$$(t+4)(t-4) - 4(t) = -16$$

$$t^2 - 16 - 4t = -16$$

$$+16$$

$$+16$$

$$t^2 - 4t = 0$$

$$t(t-4) = 0$$

$$\cancel{t=0} \quad \cancel{t=4}$$

No Solution

$$F. \left(\frac{4x+1}{x+1} = \frac{12}{x^2-1} + 3 \right) (x+1)(x-1)$$

$$(4x+1)(x-1) = 12 + 3(x^2-1)$$

$$4x^2 - 4x + x - 1 = 12 + 3x^2 - 3$$

$$4x^2 - 3x - 1 = 3x^2 + 9 - 3x^2 - 9 - 3x^2 - 9$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x=5 \quad x=-2$$

$$G. \frac{1}{x-2} + \frac{x-3}{7-x} = \frac{x+1}{-x^2+9x-14}$$

$$\left(\frac{1}{x-2} - \frac{x-3}{x-7} = -\frac{x+1}{(x-7)(x-2)} \right)$$

$$1(x-7) - (x-3)(x-2) = -(x+1)$$

$$x-7 - (x^2-5x+6) = -x-1$$

$$x-7 - x^2+5x-6 = -x-1$$

$$+13 + x^2 - 6x + x^2 - 6x + 13$$

$$0 = x^2 - 7x + 12$$

$$0 = (x-4)(x-3)$$

$$x=3$$

$$x=4$$

Precalculus Unit 3 Homework—Solving Rational Equations

Solve each equation. Be sure to check for extraneous solutions.

1. $\frac{3}{4n} + \frac{1}{n} = \frac{7}{8}$ $n=2$

2. $\frac{2}{3n} + \frac{4}{n} = \frac{7}{9}$ $n=6$

3. $\frac{6-x}{4-x} = \frac{3}{5}$ $n=9$

4. $\frac{2x-5}{8x-5} = \frac{1}{4}$ No Solution

5. $\frac{x-4}{x-2} = 2$ $x=0$

6. $\frac{2x-4}{x-2} = 3$ No Solution

7. $\frac{3}{2n+1} - \frac{6}{4n+2} = 0$ Infinitely Many
except $-\frac{1}{2}$
 $\mathbb{R} \times \neq -\frac{1}{2}$

8. $\frac{3}{2x-1} = \frac{7}{4x-2}$ No Solution

9. $\frac{3x+5}{6} - \frac{10}{x} = \frac{x}{2}$ $x=12$

10. $\frac{5}{1+y} - \frac{3}{1-y} = 2$ $y=0$ $y=4$

11. $\frac{1}{y-3} = \frac{6}{y^2-9}$ No Solution

12. $\frac{n-2}{n} - \frac{n-3}{n-6} = \frac{1}{n}$ $n=3$

13. $\frac{1}{2x-1} - \frac{3}{4x^2-1} = 0$ $x=1$

14. $\frac{1}{y^2-y} + \frac{1}{1-y} = \frac{1}{2}$ $y=-2$

15. $\frac{1}{x-4} + \frac{2}{x^2-16} = \frac{3}{x+4}$ $x=9$

16. $\frac{3a}{a-1} - \frac{4}{a+1} = \frac{4}{a^2-1}$ $a=0$
 $a=\frac{1}{3}$

17. $\frac{x-2}{x^2-x-6} = \frac{1}{x^2-4} + \frac{3}{2x+4}$
 $x=4$ $x=1$

18. $\frac{x}{x+1} - \frac{x+1}{x-4} = \frac{5}{x^2-3x-4}$
No Solution