

Precalculus Unit 2

Notes—Solving Quadratic Polynomials

There are 4 methods to solve quadratic equations algebraically:

- Factoring
- Extracting square roots
- Completing the square
- Using the quadratic formula

Example 1 Use factoring to solve each quadratic equation:

a)  $5x^2 - 45 = 0$   
 $\frac{5}{5} \quad \frac{-45}{-45}$   
 $x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x+3=0 \quad x-3=0$   
 $\frac{-3}{-3} \quad \frac{+3}{+3}$

$x = -3 \quad x = 3$

b)  $10x^2 + 13x = 3$

$10x^2 + 13x - 3 = 0$

$x^2 + 13x - 30 = 0$

$(x+15)(x-2) = 0$

$(x+\frac{3}{2})(x-\frac{1}{5}) = 0$

$x = -\frac{3}{2} \quad x = \frac{1}{5}$

c)  $8x^2 - 2x - 18 = -15$   
 $\frac{+15}{+15} \quad \frac{+15}{+15}$

$8x^2 - 2x - 3 = 0$

$(4x-3)(2x+1) = 0$

$x = \frac{3}{4} \quad x = -\frac{1}{2}$

$x^2 - 2x - 3 = 0$

$x^2 - 2x - 24 = 0$

$(x-\frac{6}{8})(x+\frac{4}{8}) = 0$

$x = \frac{6}{8} \quad x = -\frac{4}{8}$

$x = \frac{3}{4} \quad x = -\frac{1}{2}$

d)  $6x^2 + 3x - 3 = 0$   
 $\frac{3}{3} \quad \frac{-3}{-3}$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

$x = \frac{1}{2} \quad x = -1$

$2x^2 + x - 1 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \quad x = 1$

$x = -1 \quad x = \frac{1}{2}$

Example 2 Use the square root method to solve each quadratic equation:

a)  $\sqrt{(5x-1)^2} = \sqrt{12}$   
 $5x-1 = \pm 2\sqrt{3}$   
 $\frac{+1}{+1} \quad \frac{+1}{+1}$

$\frac{5x}{5} = \frac{1 \pm 2\sqrt{3}}{5}$

$x = \frac{1 \pm 2\sqrt{3}}{5}$

b)  $\sqrt{(x+3)^2} = \sqrt{18}$

$x+3 = \pm 3\sqrt{2}$   
 $\frac{-3}{-3} \quad \frac{-3}{-3}$

$x = -3 \pm 3\sqrt{2}$

c)  $x^2 + 10x + 25 = 121$

$\sqrt{(x+5)^2} = \sqrt{121}$

$x+5 = \pm 11$   
 $\frac{-5}{-5} \quad \frac{-5}{-5}$

$x = -5 \pm 11$

$x = 6$   
 $x = -16$

d)  $x^2 + 6x + 9 = 8$   
 $\frac{-9}{-9} \quad \frac{-9}{-9}$

$x^2 + 6x = -1$   
 $\frac{+9}{+9} \quad \frac{+9}{+9}$

$\sqrt{(x+3)^2} = \sqrt{8}$

$x+3 = \pm 2\sqrt{2}$   
 $\frac{-3}{-3} \quad \frac{-3}{-3}$

$x = -3 \pm 2\sqrt{2}$

Example 3 Use completing the square to solve each quadratic equation:

a)  $x^2 + 2x - 14 = 0$

$$x^2 + 2x = 14$$

$$\sqrt{(x+1)^2} = \sqrt{15}$$

$$x+1 = \pm\sqrt{15}$$

$$\begin{matrix} -1 & -1 \\ -1 & -1 \end{matrix}$$

$$x = -1 \pm \sqrt{15}$$

b)  $2x^2 - 8x - 13 = 7$

$$\frac{2x^2 - 8x}{2} = \frac{20}{2}$$

$$x^2 - 4x + 4 = 10 + 4$$

$$\sqrt{(x-2)^2} = \sqrt{14}$$

$$x-2 = \pm\sqrt{14}$$

$$\begin{matrix} +2 & +2 \\ +2 & +2 \end{matrix}$$

$$x = 2 \pm \sqrt{14}$$

c)  $-x^2 - 2x + 5 = 0$

$$x^2 + 2x - 5 = 0$$

$$x^2 + 2x + 1 = 5 + 1$$

$$\sqrt{(x+1)^2} = \sqrt{6}$$

$$x+1 = \pm\sqrt{6}$$

$$x = -1 \pm \sqrt{6}$$

d)  $-2x^2 + 6x + 9 = 0$

$$\frac{-2x^2 + 6x}{-2} = \frac{-9}{-2}$$

$$x^2 - 3x + \frac{9}{4} = \frac{9}{2} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{18}{4} + \frac{9}{4}$$

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{\frac{27}{4}}$$

$$x - \frac{3}{2} = \pm \frac{3\sqrt{3}}{2}$$

$$\begin{matrix} +\frac{3}{2} & +\frac{3}{2} \\ +\frac{3}{2} & +\frac{3}{2} \end{matrix}$$

$$x = \frac{3 \pm 3\sqrt{3}}{2}$$

Quadratic Formula:



If  $ax^2 + bx + c = 0$  where  $a \neq 0$  then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 4 Use the quadratic formula to solve each quadratic equation:

a)  $x^2 + 2x - 14 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-14)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{60}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{15}}{2}$$

$$x = -1 \pm \sqrt{15}$$

b)  $2x^2 - 8x - 13 = 7$

$$x = \frac{-(-8) \pm \sqrt{64 - 4(2)(-20)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{224}}{4}$$

$$x = \frac{8 \pm 4\sqrt{14}}{4}$$

$$x = 2 \pm \sqrt{14}$$

$$2x^2 - 8x - 20 = 0$$

$$x^2 - 4x - 10 = 0$$

$$x = \frac{-(-4) \pm \sqrt{16 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{56}}{2}$$

$$x = \frac{4 \pm 2\sqrt{14}}{2}$$

$$x = 2 \pm \sqrt{14}$$

Precalculus Unit 2

Homework—Solving Quadratic Functions

Solve the quadratic equation by factoring.

1.  $6x^2 + 3x = 0$   
 $3x(2x+1) = 0$   
 $x = 0 \quad x = -\frac{1}{2}$

3.  $x^2 - 10x + 9 = 0$   
 $(x-9)(x-1) = 0$   
 $x = 9 \quad x = 1$

5.  $3 + 5x - 2x^2 = 0$   
 $-2x^2 + 5x + 3 = 0$   
 $2x^2 - 5x - 3 = 0$   
 $(2x+1)(x-3) = 0$   
 $x = -\frac{1}{2} \quad x = 3$

2.  $x^2 - 2x - 8 = 0$   
 $(x-4)(x+2) = 0$   
 $x = 4 \quad x = -2$

4.  $16x^2 + 56x + 49 = 0$   
 $(4x+7)(4x+7) = 0$   
 $x = -\frac{7}{4} \quad x = -\frac{7}{4}$

6.  $2x^2 = 19x + 33$   
 $2x^2 - 19x - 33 = 0$   
 $(2x+3)(x-11) = 0$   
 $x = -\frac{3}{2} \quad x = 11$

Solve the equation by extracting square roots.

7.  $\sqrt{x^2} = \sqrt{27}$   
 $x = \pm 3\sqrt{3}$

9.  $\sqrt{(x-7)^2} = \sqrt{18}$   
 $x-7 = \pm 3\sqrt{2}$   
 $+7 \quad +7$   
 $x = 7 \pm 3\sqrt{2}$

8.  $\frac{3x^2}{8} = \frac{36}{3}$   
 $\sqrt{x^2} = \sqrt{12}$   
 $x = \pm 2\sqrt{3}$

10.  $\sqrt{(x+4)^2} = \sqrt{20}$   
 $x+4 = \pm 2\sqrt{5}$   
 $-4 \quad -4$   
 $x = -4 \pm 2\sqrt{5}$

Solve the quadratic equation by completing the square.

11.  $x^2 + 4x - 32 = 0$   
 $x^2 + 4x + 4 = 32 + 4$   
 $\sqrt{(x+2)^2} = \sqrt{36}$   
 $x+2 = \pm 6$   
 $-2 \quad -2$   
 $x = -2 \pm 6$   
 $x = -8 \quad x = 4$

12.  $x^2 - 2x - 5 = 0$   
 $x^2 - 2x + 1 = 5 + 1$   
 $\sqrt{(x-1)^2} = \sqrt{6}$   
 $x-1 = \pm\sqrt{6}$   
 $+1 \quad +1$   
 $x = 1 \pm \sqrt{6}$

13.  $x^2 + 8x + 14 = 0$   
 $x^2 + 8x + 16 = -14 + 16$   
 $\sqrt{(x+4)^2} = \sqrt{2}$   
 $x+4 = \pm\sqrt{2}$   
 $x = -4 \pm \sqrt{2}$

14.  $x^2 - 3x - 7 = 0$   
 $x^2 - 3x + \frac{9}{4} = 7 + \frac{9}{4}$   
 $\sqrt{(x-\frac{3}{2})^2} = \sqrt{\frac{37}{4}}$   
 $x-\frac{3}{2} = \pm\sqrt{\frac{37}{4}}$   
 $x = \frac{3}{2} \pm \frac{\sqrt{37}}{2}$

15.  $x^2 + 5x - 3 = 0$   
 $x^2 + 5x + \frac{25}{4} = 3 + \frac{25}{4}$   
 $\sqrt{(x+\frac{5}{2})^2} = \sqrt{\frac{37}{4}}$   
 $x+\frac{5}{2} = \pm\sqrt{\frac{37}{4}}$   
 $x = -\frac{5}{2} \pm \sqrt{\frac{37}{4}}$   
 $x = \frac{-5 \pm \sqrt{37}}{2}$

16.  $x^2 - 7x - 3 = 0$   
 $x^2 - 7x + \frac{49}{4} = 3 + \frac{49}{4}$   
 $\sqrt{(x-\frac{7}{2})^2} = \sqrt{\frac{61}{4}}$   
 $x-\frac{7}{2} = \pm\sqrt{\frac{61}{4}}$   
 $+ \frac{7}{2} \quad + \frac{7}{2}$   
 $x = \frac{7 \pm \sqrt{61}}{2}$