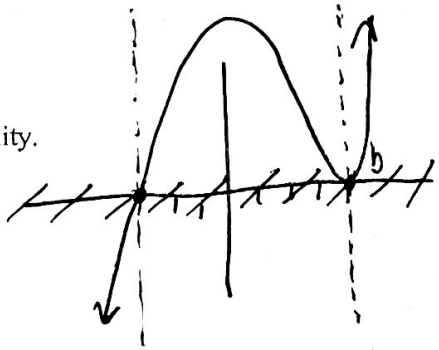
When you solve an inequality, a solution is any value that will make the statement true.



sections Not Asymptotes

Example 1 Solve each equation or inequality.

A. $(x+3)(x^2+1)(x-4)^2=0$

$x=-3 \quad x=\pm i \quad x=4$ M2

B. $(x+3)(x^2+1)(x-4)^2>0$

$(-3,4) \cup (4,\infty)$

C. $(x+3)(x^2+1)(x-4)^2<0$

$(-\infty,-3)$

D. $(x+3)(x^2+1)(x-4)^2\geq 0$

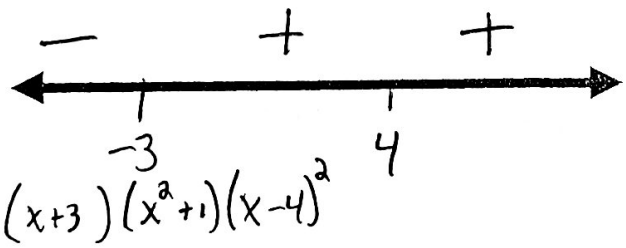
$[-3,\infty)$

E. $(x+3)(x^2+1)(x-4)^2\leq 0$

$(-\infty,-3] \cup [4]$

METHOD #1:

THE SIGN CHART



******Note---ALL of the x-intercepts of the polynomial are placed on the sign chart******

METHOD #2:

THE GRAPH

Example 2 Solve each inequality.

A. $(x^2+7)(3x^2+1)\geq 0$

$(-\infty,\infty)$

B. $(x^2+7)(3x^2+1)\leq 0$

$x^2+7=0$ $\quad 3x^2+1=0$

$x^2=-7$ $\quad x^2=-\frac{1}{3}$

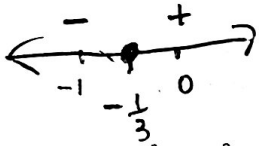
No zeros $\quad$ No zeros

No Solution

C. $(x^2+x+7)(3x+1)\geq 0$

$x=\frac{-1\pm\sqrt{1-4(1)(7)}}{2(1)}=\frac{-1\pm\sqrt{-27}}{2}$

$x=-\frac{1}{3}$ $\quad [-\frac{1}{3},\infty)$



D. $(x^2+x+7)(3x+1)>0$

$(-\frac{1}{3},\infty)$

E. $2x^3-5x^2\leq 18x-45$

$2x^3-5x^2-18x+45\leq 0$

$x^2(2x-5)-9(2x-5)\leq 0$

$(x^2-9)(2x-5)\leq 0$

$(x+3)(x-3)(2x-5)\leq 0$

$(-\infty,-3]\cup[2.5,3]$

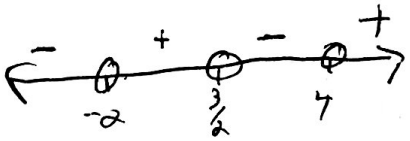
F. $2x^3-7x^2-10x+24>0$

4	2	−7	−10	24
		8	4	−24
	2	1	−6	0

$2x^2+x-6=0$

$(2x-3)(x+2)=0$

$(x-4)(2x-3)(x+2)>0$

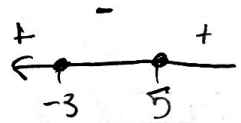


$(-2,\frac{3}{2})\cup(4,\infty)$

G. $x^2-2x\geq 15$

$x^2-2x-15\geq 0$

$(x-5)(x+3)\geq 0$



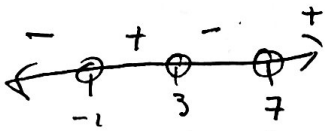
$(-\infty,-3]\cup[5,\infty)$

H. $x^3-9x^2+11x+21<0$

−1	1	−9	11	21
		−1	10	−21
	1	−10	21	0

$x^2-10x+21=0$

$(x-3)(x-7)(x+1)<0$



$(-\infty,-1)\cup(3,7)$

When you solve a rational inequality, you must include all of the zeros AND undefined values for the function on the sign chart.

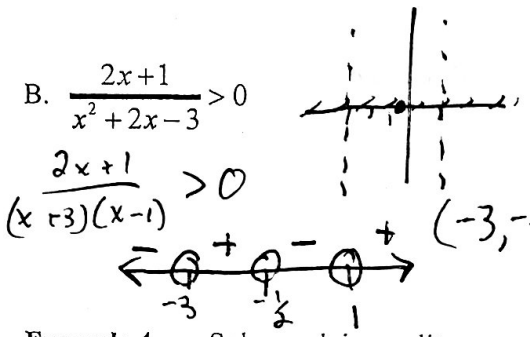
Example 3 Solve each equation or inequality.

A. $\frac{2x+1}{x^2+2x-3}=0$ $\quad x^2+2x-3 \neq 0$ $\rightarrow$ $2x+1=0$

$x=-\frac{1}{2}$

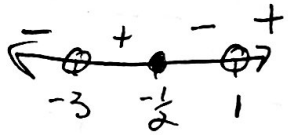
B. $\frac{2x+1}{x^2+2x-3}>0$

$\frac{2x+1}{(x+3)(x-1)}>0$



$(-3,-\frac{1}{2}) \cup (1,\infty)$

C. $\frac{2x+1}{x^2+2x-3} \leq 0$



$(-\infty,-3) \cup [-\frac{1}{2},1)$

Example 4 Solve each inequality.

A. $\frac{5}{x+3} < -\frac{3}{x+1}$

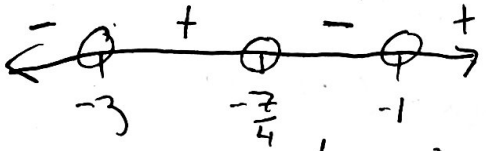
$\frac{5}{x+3}+\frac{3}{x+1}<0$

$\frac{5}{x+3}\cdot\frac{x+1}{x+1}+\frac{3}{x+1}\cdot\frac{x+3}{x+3}<0$

$\frac{5x+5+3x+9}{(x+3)(x+1)}<0$

$\frac{8x+14}{(x+3)(x+1)}<0$

$x \neq -3 \quad x \neq -1$ $\quad x=\frac{-14}{8}=-\frac{7}{4}$



$(-\infty,-3) \cup (-\frac{7}{4},-1)$

B. $\frac{5x+1}{2x^2} > \frac{9x+5}{4x^2+2x}$

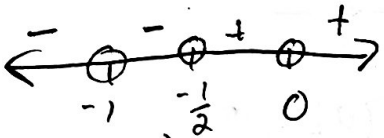
$\frac{2x+1}{2x+1}\cdot\frac{5x+1}{2x^2}-\frac{9x+5}{2x(2x+1)}\cdot\frac{x}{x}>0$

$\frac{10x^2+2x+5x+1-(9x^2+5x)}{2x^2(2x+1)}>0$

$\frac{x^2+2x+1}{2x^2(2x+1)}>0$

$\frac{(x+1)(x+1)}{2x^2(2x+1)}>0$

$x \neq 0 \quad x \neq -\frac{1}{2} \quad x=-1$

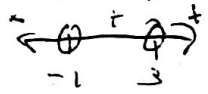


$(-\frac{1}{2},0) \cup (0,\infty)$

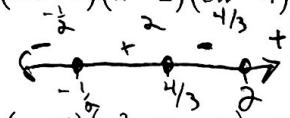
Precalculus Unit 3 Homework—Solving Polynomial & Rational Inequalities

In exercises 1-6, complete the factoring if needed, and solve the polynomial inequality.

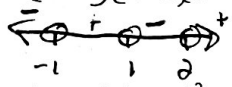
1. $(x+1)(x-3)^2 > 0$ $(-1,3) \cup (3,\infty)$



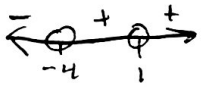
2. $(2x+1)(x-2)(3x-4) \le 0$ $(-\infty, -\frac{1}{2}] \cup [\frac{4}{3}, 2]$



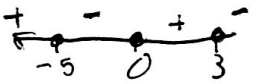
3. $(x+1)(x^2-3x+2) < 0$
$(x+1)(x-2)(x-1) < 0$ $(-\infty,-1) \cup (1,2)$



4. $(x+4)(x-1)^2 > 0$ $(-4,1) \cup (1,\infty)$



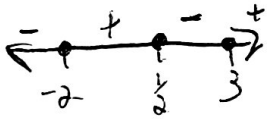
5. $-x^3 - 2x^2 + 15x \le 0$
$-x(x^2+2x-15) \le 0$
$-x(x+5)(x-3) \le 0$
$0, -5, 3$ $[-5,0] \cup [3,\infty)$



6. $2x^3 - 3x^2 - 11x + 6 \ge 0$

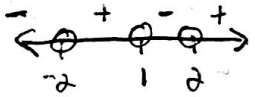
2	2	-3	-11	6
		4	14	-6
	2	-7	3	0

$(x+2)(2x^2-7x+3) \ge 0$
$(x+2)(2x-1)(x-3) \ge 0$
$-2, \frac{1}{2}, 3$ $[-2, \frac{1}{2}] \cup [3,\infty)$

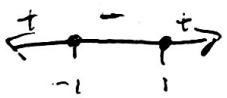


In exercises 7-11, solve each inequality.

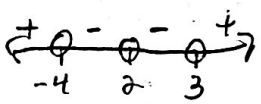
7. $\frac{x-1}{x^2-4} < 0$
$\frac{x-1}{(x+2)(x-2)} < 0$ $(-\infty,-2) \cup (1,2)$



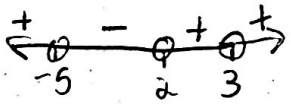
8. $\frac{x^2-1}{x^2+1} \le 0$
$\frac{(x+1)(x-1)}{x^2+1} \le 0$ $[-1,1]$



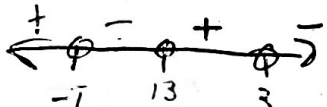
9. $\frac{x^2+x-12}{x^2-4x+4} > 0$
$\frac{(x+4)(x-3)}{(x-2)(x-2)}$ $(-\infty,-4) \cup (3,\infty)$



10. $\frac{x^2+3x-10}{x^2-6x+9} < 0$
$\frac{(x+5)(x-2)}{(x-3)(x-3)} < 0$ $(-5,2)$



11. $\frac{x-5}{x+1} < \frac{x+2}{x-3}$
$\frac{x-3}{x-3} \cdot \frac{x-5}{x+1} - \frac{x+2}{x-3} \cdot \frac{x+1}{x+1} < 0$
$\frac{x^2-5x-3x+15-(x^2+x+2x+2)}{(x-3)(x+1)} < 0$
$\frac{-11x+13}{(x-3)(x+1)} < 0$
$+\frac{13}{11}, 3, -1$



$(-1, \frac{13}{11}) \cup (3,\infty)$