Precalculus *Unit 2* *Notes—Real Zeros of Polynomials*

Remember long division? How about with polynomials?

$$\frac{1373}{3}$$ $$\frac{3x^2-7x+2}{x+5}$$

$457\frac{2}{3}$

$3\overline{)1373}$

-12

17

-15

23

-21

2

$457+\frac{2}{3}$

$3x-22+\frac{112}{x+5}$

$x+5\overline{)3x^2-7x+2}$

$-(3x^2+15x)$

$-22x+2$

$-(-22x-110)$

112

STEP#1: Determine what you can multiply the 1st term in **DIVISOR** by to get as close to the first term of the **DIVIDEND** as possible.

STEP#2: Multiply the whole **DIVISOR** by that amount

STEP #3: Subtract

STEP#4: Bring down next term

STEP#5: REPEAT

**** Note: Write your final answer as the *QUOTIENT* + $\frac{REMAINDER}{DIVISOR}$

Example 1 Divide: $\frac{2x^4-x^3-2}{2x^2+x+1}$

$x^2-x+\frac{x-2}{2x^2+x+1}$

$2x^2+x+1\overline{)2x^4-x^3+0x^2+0x-2}$

$-(2x^4+x^3+1x^2)$

$-2x^3-1x^2+0x$

$-(-2x^3-x^2-x)$

$x-2$

$$\boxed{x^2-x+\frac{x-2}{2x^2+x+1}}$$

Place holders are NECESSARY when you are "missing" terms.

The following statements are all equivalent:

- $x=c$ is a solution (or root) of the equation $f(x)=0$.
- When $f(x)$ is divided by $(x-c)$, the remainder equals 0.
- c is a zero of the function $f(x)$.
- c is an x-intercept of the graph of $f(x)$ if c is a real number.
- $(x-c)$ is a factor of $f(x)$.

You get to use *SYNTHETIC DIVISION* whenever your **DIVISOR** is linear (in the form $mx+b$)

Example 2 Find the quotient using both long and synthetic division: $\dfrac{2x^3-3x^2-5x-12}{x-3}$

LONG DIVISION

$$\begin{array}{r} 2x^2+3x+4 \\ x-3\overline{)\,2x^3-3x^2-5x-12} \\ -(2x^3-6x^2) \\ \hline 3x^2-5x \\ -(3x^2-9x) \\ \hline 4x-12 \\ -(4x-12) \\ \hline 0 \end{array}$$

x-3 is a factor of numerator

SYNTHETIC DIVISION

$$\begin{array}{c|cccc} 3 & 2 & -3 & -5 & -12 \\ & & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 & R0 \\ & x^2 & x & \text{Constant} & \end{array}$$

$2x^2+3x+4$

We use long and synthetic division mainly to find zeros of polynomials that we could not factor using any of the methods that we have learned before.

If the process of long/synthetic division is "embedded" in a problem with a polynomial use the RATIONAL ROOT THEOREM to know what to divide by (since it will not be so specific).

THEOREM: To identify all ***POSSIBLE RATIONAL ZEROS*** (NOT definite zeros), we begin by listing all the factors of the constant term, factors of our leading coefficient, and then we create the list of possibilities to use by finding ALL COMBINATIONS of these factors using the ones from the constant term as "numerators" and factors from leading coefficient as "denominators." Then we simplify all of these numbers, eliminate repeats, and add "plus or minus" to each of them.

Example 3 List all possible rational zeros of $f(x)=3x^4+2x^3-7x+6$

STEP #1: List factors of constant------------------→ $\pm1, \pm2, \pm3, \pm6$

STEP #2: List factors of leading coefficient-----→ $\pm1, \pm3$

STEP #3: Write out all combos using factors of Constant as "numerators" & factors of the Leading Coefficient as "denominators" ---------→

STEP #4: Simplify ALL, eliminate repeats & add "plus or minus" to each--------→ $\boxed{\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{3}, \pm\frac{2}{3}}$

Example 4 Determine if $f(x) = x^3 - 3x^2 + 1$ has any rational zeros.

$$\frac{\pm 1}{\pm 1} \quad \frac{\text{constant}}{\text{L.C.}}$$

$$\begin{array}{r|rrrr} \cancel{1} & 1 & -3 & 0 & 1 \\ & & 1 & -2 & -2 \\ \hline & 1 & -2 & -2 & -1 \end{array}$$

$$\begin{array}{r|rrrr} \cancel{-1} & 1 & -3 & 0 & 1 \\ & & -1 & 4 & -4 \\ \hline & 1 & -4 & 4 & -3 \end{array}$$

No Rational Zeros

Example 5 Find all of the zeros for each polynomial:

a) $f(x) = 3x^3 + 4x^2 - 5x - 2$ $\quad \frac{\pm 1 \ \pm 2}{\pm 1 \ \pm 3}$

$$\begin{array}{r|rrrr} 1 & 3 & 4 & -5 & -2 \\ & & 3 & 7 & 2 \\ \hline & 3 & 7 & 2 & 0 \end{array}$$

$3x^2 + 7x + 2 = 0$

$x^2 + 7x + 6 = 0$

$(x + \frac{6}{3})(x + \frac{1}{3}) = 0$

$x = -2 \quad x = -\frac{1}{3}$

$x = 1$

$x = -2$

$x = -\frac{1}{3}$

b) $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$

$$\begin{array}{r|rrrrr} 4 & 2 & -7 & -8 & 14 & 8 \\ & & 8 & 4 & -16 & -8 \\ \hline & 2 & 1 & -4 & -2 & 0 \end{array}$$

$2x^3 + x^2 / - 4x - 2 = 0$

$x^2(2x + 1) - 2(2x + 1) = 0$

$(x^2 - 2)(2x + 1) = 0$

$x^2 - 2 = 0 \qquad 2x + 1 = 0$

$+2 \quad +2 \qquad -1 \quad -1$

$\sqrt{x^2} = \sqrt{2} \qquad \frac{2x}{2} = -\frac{1}{2}$

$x = \pm\sqrt{2} \qquad x = -\frac{1}{2}$

$\boxed{x = 4 \quad x = -\frac{1}{2} \quad x = \sqrt{2} \quad x = -\sqrt{2}}$

c) $f(x) = 2x^4 - 5x^3 - 2x^2 + 11x - 6$ $\quad \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1 \ \pm 2}$

$$\begin{array}{r|rrrrr} 1 & 2 & -5 & -2 & 11 & -6 \\ & & 2 & -3 & -5 & 6 \\ \hline 1 & 2 & -3 & -5 & 6 & 0 \\ & & 2 & -1 & -6 & \\ \hline & 2 & -1 & -6 & 0 & \end{array}$$

$2x^2 - x - 6 = 0$

$x^2 - x - 12 = 0$

$(x - \frac{4}{2})(x + \frac{3}{2}) = 0$

$x = 2 \quad x = -\frac{3}{2}$

$\boxed{x = 1 \quad x = 1 \quad x = 2 \quad x = -\frac{3}{2}}$

Precalculus Unit 2 *Homework—Real Zeros of Polynomials*

In exercises 1-6, use the Factor Theorem to determine whether the first polynomial is a factor of the second polynomial.

1. $x-1$; x^3-x^2+x-1 yes
2. $x-3$; x^3-x^2-x-15 yes
3. $x-2$; x^3+3x-4 No
4. $x-2$; x^3-3x-2 yes
5. $x+2$; $4x^3+9x^2-3x-10$ yes
6. $x+1$; $2x^{10}-x^9+x^8+x^7+2x^6-3$ No

3 | 1 -1 -1 -15
3 6 15
1 2 5 0

2 | 1 0 3 -4
2 4 14
1 2 7

2 | 1 0 -3 -2
2 4 2
1 2 1 0

-2 | 4 9 -3 -10
-8 -2 10
4 1 -5 0

-1 | 2 -1 1 1 2 0 0 0 0 0 -3
-2 3 -4 3 -5
2 -3 4 -3 5 0

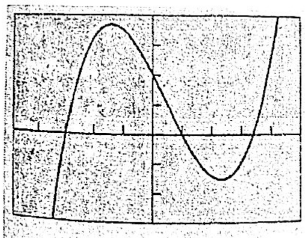
In exercises 7-8, use the graph to guess possible linear factors of $f(x)$. Then completely factor $f(x)$.

7. $f(x)=5x^3-7x^2-49x+51$

-3 | 5 -7 -49 51
-15 66 -51
5 -22 17 0

$(x+3)\,5x^2-22x+17=0$

$f(x)=(x+3)(5x-17)(x-1)$



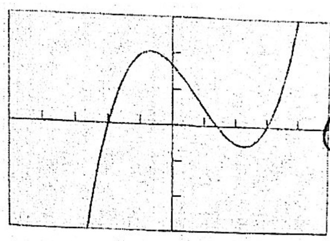
[-5, 5] by [-75, 100]

8. $f(x)=5x^3-12x^2-23x+42$

-2 | 5 -12 -23 42
-10 44 -42
5 -22 21 0

$(x+2)(5x^2-22x+21)=0$

$(x+2)(5x-7)(x-3)=f(x)$



[-5, 5] by [-75, 75]

In exercises 9-11, use the Rational Root Theorem to write a list of all potential rational zeros. Then determine which ones, if any, are zeros.

9. $f(x)=6x^3-5x-1$ $\frac{\pm 1}{\pm 1, \pm 2, \pm 3, \pm 6}$

10. $f(x)=3x^3-7x^2+6x-14$ $\longrightarrow$ $\frac{\pm 1, \pm 2, \pm 7, \pm 14}{\pm 1 \ \pm 3}$

11. $f(x)=2x^3-x^2-9x+9$ $\frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2}$

In exercises 12-14, find all of the real zeros of the function, finding exact values whenever possible. Identify each zero as rational or irrational.

12. $f(x)=2x^3-3x^2-4x+6$

13. $f(x)=x^3+x^2-8x-6$

14. $f(x)=2x^4-7x^3-2x^2-7x-4$