Precalculus Unit 2 Notes—Polynomial Functions

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*****Graphing Polynomial Functions*****

Not only are graphs of polynomials unbroken without jumps or holes, but they are *smooth* lines or curves, with no sharp corners or cusps.

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Investigation 1 - End Behavior:

1. Using a graphing calculator, sketch the following functions:

   a. $y = 3x^4 - 7x^3 + x^2 + 9$

   b. $y = -1/2x^6 - 4x^5 + 2x^3 - 11x + 5$

   c. $y = 2x^3 + 5x^2 - 3x + 1$

   d. $y = -3x^5 + 7x^3 - 5$

2. What affects the right end behavior?

   $\pm a$

3. What affects the left end behavior?

highest degree is even or odd

THEOREM ----A polynomial function of degree n has at most n-1 local extrema and at most n zeros.

End Behavior of Polynomial Functions

In order to determine the end behavior of a polynomial function you need only 2 pieces of information:

1st: You must know the degree of the polynomial. If the degree is even the LEFT END BEHAVIOR (L.E.B) & the RIGHT END BEHAIVOR (R.E.B) will be the same. If the degree is odd then the L.E.B. and the R.E.B. will be opposite.

2nd: You must know the sign of the leading coefficient (L.C.) of the polynomial. If the L.C. is POSITIVE then the R.E.B. will be: $\lim_{x\to\infty} f(x) = \infty$. However, if the L.C. is NEGATIVE then the R.E.B. will be: $\lim_{x\to\infty} f(x) = -\infty$.

Zeros of Polynomial Functions

Recall that finding the real-number zeros of a function f is equivalent to finding the x-intercepts of the graph of $y = f(x)$ or the solutions to the equation $f(x) = 0$.

Zeros, solutions, roots, x-intercepts

Investigation 2 - Local Extrema and Zeros

1. Using a graphing calculator, investigate various third degree polynomial functions to see how many extrema and how many zeros the function can have.

2. Using a graphing calculator, investigate various fourth degree polynomial functions to see how many extrema and how many zeros the function can have.

⇨ **In general, if your degree is "n", how many extrema are possible?** $n-1$

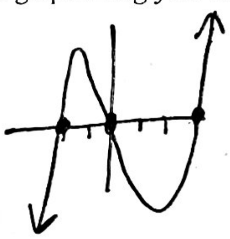
⇨ **In general, if your degree is "n", how many zeros are possible?** n

Example 1 Find the zeros of $f(x) = x^3 - x^2 - 6x$ and then sketch the graph using your knowledge of intercepts and end behavior.

$f(x) = x(x^2 - x - 6)$

$f(x) = x(x+2)(x-3)$

$x = 0 \quad x = -2 \quad x = 3$



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***** Multiplicity of a Zero of a Polynomial Function *****

If f is a polynomial function & $(x - c)^m$ is a factor of f then c is a zero of **multiplicity** m of f.

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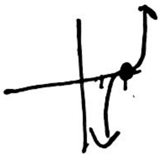
$x=2$

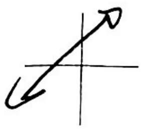
$(x-2)^3$ $(2,0)$ $x = 2$ Multiplicity 3 $y = (x-3)^2$

Zeros of Odd and Even Multiplicity

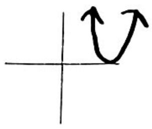


If a polynomial function f has a real zero c of odd multiplicity, then the graph of f crosses the x-axis at $(c, 0)$ and the value of f changes sign at $x = c$. If a polynomial function f has a real zero c of even multiplicity, then the graph of f does not cross the x-axis at $(c, 0)$ and the value of f does not change sign at $x = c$.

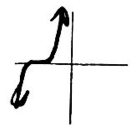
⇨ If the multiplicity of a zero is 1, it will Cross/Pass the x-axis in the typical "straight through" manner.



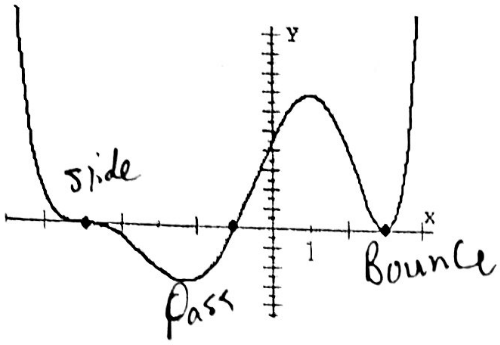
⇨ If the multiplicity of a zero is EVEN, it will bounce at the x-axis & will NOT cross through.



⇨ If the multiplicity of a zero is greater than 1 & ODD, it will Slide at the x-axis & will cross through.



Example 2 Identify the zeros of the polynomial function:



Multiplicity of 1: $x = -1$

Even Multiplicity: $x = 3$ Multiplicity of 2

Odd Multiplicity > 1: $x = -5$ Multiplicity of 3

Example 3 State the degree and find the zeros of each function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of each polynomial.

(a) $f(x) = (x+2)^3(x-1)^2$

$\boxed{y\text{-int}: 8}$ Degree: 5

$x = -2$ Mult. 3

$x = 1$ Mult. 2

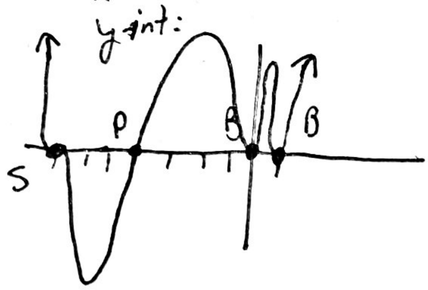
(b) $f(x) = x^2(x+7)^3(x-1)^4(x+4)$

Degree: 10

$x = 0$ M: 2 $x = 1$ M 4

$x = -7$ M: 3 $x = -4$ M 1

y-int:

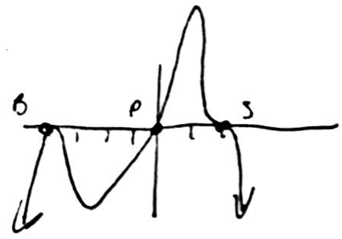


(c) $f(x) = -x(x+4)^2(x-2)^3$

Degree: 6

$x = 0$ M 1 $x = -4$ M 2 $x = 2$ M 3

y-int: 0



Example 4 Describe how to transform the graph of a basic monomial function into the graph of the given polynomial.

a) $f(x) = 4(x+1)^3$ $= x^3$

vstretch by 4

← 1

b) $g(x) = -(x-2)^4 + 5$

reflect over x

→ 2 ↑ 5

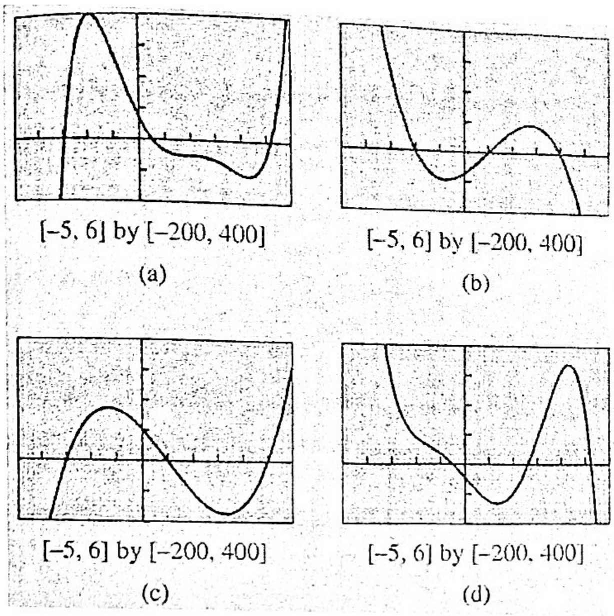
c) $h(x) = 2(x-3)^5 - 1$ $= x^5$

vert stretch of 2

→ 3 ↓ 1

Precalculus Unit 2 *Homework—Polynomial Functions*

In exercises 1-4, match the polynomial function with its graph.



[–5, 6] by [–200, 400] (a)

[–5, 6] by [–200, 400] (b)

[–5, 6] by [–200, 400] (c)

[–5, 6] by [–200, 400] (d)

1. $f(x)=7x^3-21x^2-91x+104$

C

2. $f(x)=-9x^3+27x^2+54x-73$

B

3. $f(x)=x^5-8x^4+9x^3+58x^2-164x+69$

A

4. $f(x)=-x^5+3x^4+16x^3-2x^2-95x-44$

D

Using limits, describe the end behavior of each function:

5. $f(x)=(x-1)(x+2)(x+3)$

$\lim_{x\to-\infty} f(x) = -\infty$

$\lim_{x\to\infty} f(x) = \infty$

6. $f(x)=-x^3+4x^2+31x-70$

$\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = -\infty$

7. $f(x)=(x-2)^2(x+1)(x-3)$

$\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = \infty$

8. $f(x)=(2x+1)(x-4)^3$

$\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = \infty$

Find the zeros algebraically:

9. $f(x)=x^2+2x-8$

$(x+4)(x-2)$

$x=-4$

$x=2$

10. $f(x)=9x^2-3x-2$

$f(x)=(3x-2)(3x+1)$

$x=\frac{2}{3}$ $x=-\frac{1}{3}$

11. $f(x)=3x^3-x^2-2x$

$f(x)=x(3x^2-x-2)$

$f(x)=x(3x+2)(x-1)$

$x=0$ $x=-\frac{2}{3}$ $x=1$

12. $f(x)=x^3-25x$

$f(x)=x(x^2-25)$

$x=0$ $x=5$ $x=-5$

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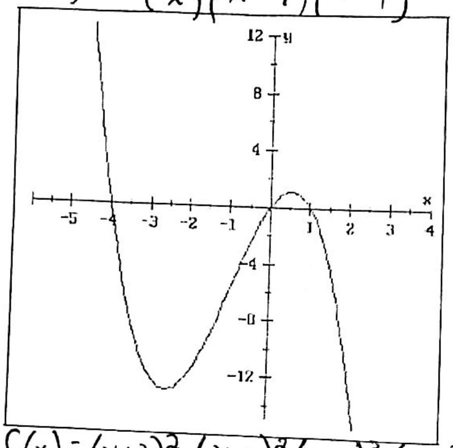
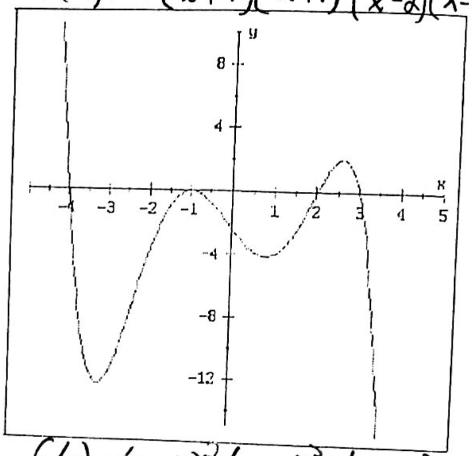
Give a possible factorization of the following polynomials. Do not multiply out factors!

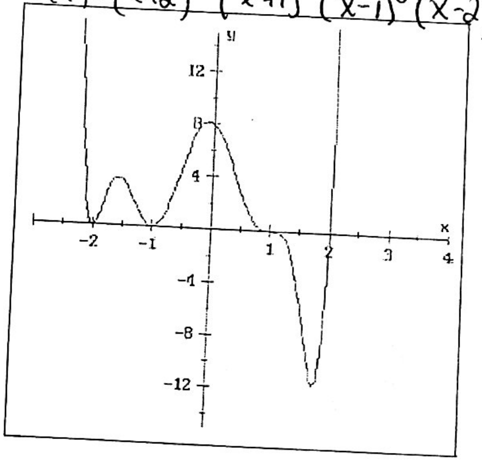
13. $f(x) = -(x)(x-1)(x+4)$



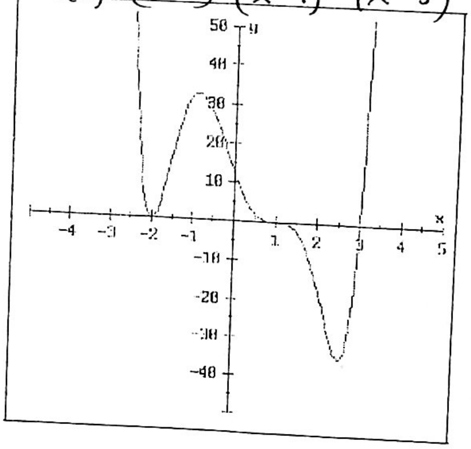
14. $f(x) = -(x+4)(x+1)^2(x-2)(x-3)$



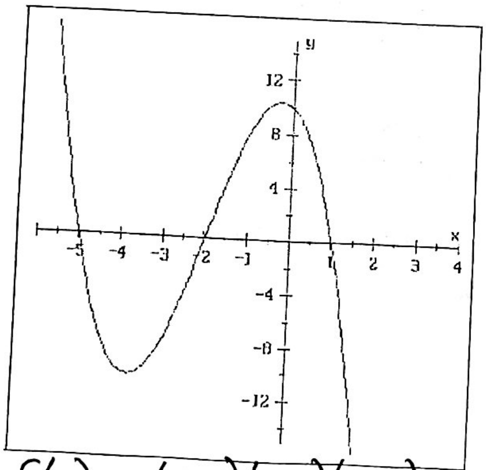
15. $f(x) = (x+2)^2(x+1)^2(x-1)^3(x-2)$



16. $f(x) = (x+2)^2(x-1)^3(x-3)$

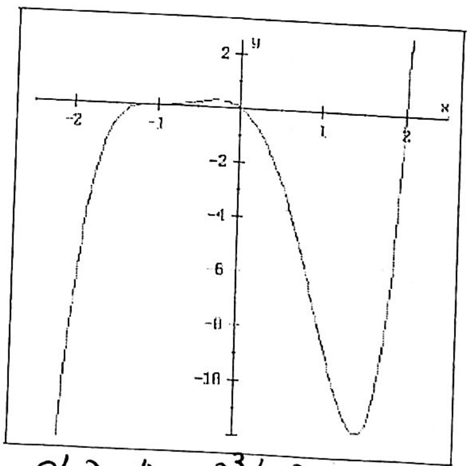


17.



$f(x) = -(x+5)(x+2)(x-1)$

18.



$f(x) = (x+1)^3(x)(x-2)$