

piecewise function-- A piecewise function is one which is defined not by a single equation, but by two or more. Each equation is valid for some interval.

Evaluating a piecewise function

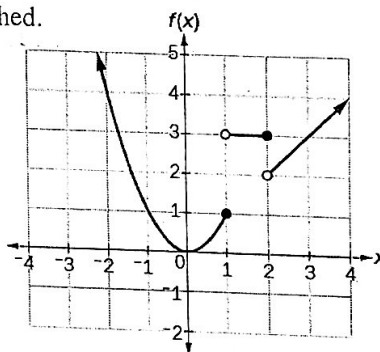
- Be sure to use the piece of the graph that contains the appropriate domain value.

Example 1 Evaluate using the piecewise function graphed.

$f(-1) = \underline{1}$

$f(1) = \underline{1}$

$f(2) = \underline{3}$



Example 2 Evaluate using the given piecewise function.

$f(2) = \underline{-6}$ $f(3) = \underline{2}$

$f(10) = \underline{93}$ $f(12) = \underline{15}$

$$f(x) = \begin{cases} -x - 4 & , \quad x < 3 \\ x^2 - 7 & , \quad 3 \leq x \leq 10 \\ \frac{120}{x} + 5 & , \quad x > 10 \end{cases}$$

Hints for graphing a piecewise function

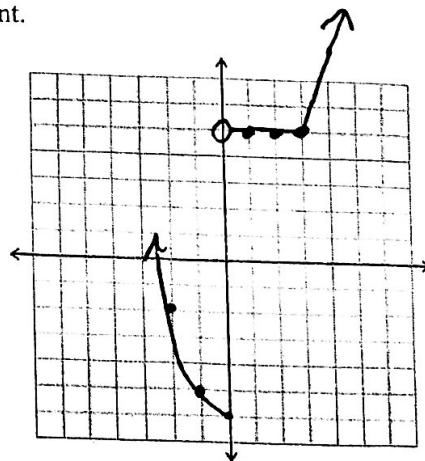
- Draw boundary lines at the "breaks". *
- Make a t-chart for each piece and graph the points.
- Use your knowledge of parent functions.
- When you have < or >, you will use an open circle at that point.
- When you have ≤ or ≥, you will use a closed circle at that point.

Example 3 Graph $f(x) = \begin{cases} x^2 - 6, & x \leq 0 \\ 5, & 0 < x \leq 3 \\ 3x - 4, & x > 3 \end{cases}$

$$\begin{array}{r|l} x^2 - 6 & \\ \hline x & 2 \\ -2 & -2 \\ -1 & -5 \\ 0 & -6 \end{array}$$

$$\begin{array}{r|l} 5 & \\ \hline x & 0 \\ 1 & 5 \\ 2 & 5 \\ 3 & 5 \end{array}$$

$$\begin{array}{r|l} 3x - 4 & \\ \hline x & 2 \\ 3 & 5 \\ 4 & 8 \\ 5 & 11 \end{array}$$



Example 4

$$\text{Graph } g(x) = \begin{cases} -\frac{1}{2}x + 2, & -5 \leq x \leq -2 \\ x^3, & -2 < x < 2 \\ -x + 10, & 2 \leq x < 6 \end{cases}$$

$$-\frac{1}{2}x + 2$$

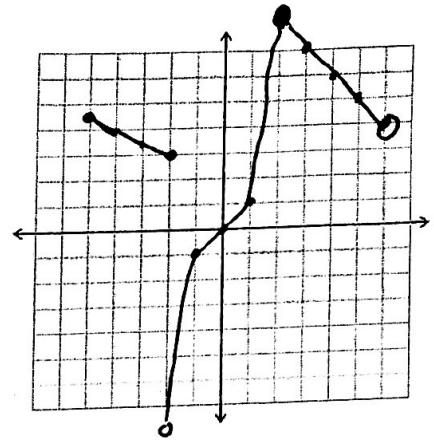
x	y
-5	4.5
-4	4
-3	3.5
-2	3

$$x^3$$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

$$-x + 10$$

x	y
2	8
3	7
4	6
5	5
6	4



Example 5

$$\text{Graph } h(x) = \begin{cases} 5, & x < -4 \\ 1 + |x|, & -4 \leq x \leq 3 \\ -\frac{2}{3}x + 6, & x > 3 \end{cases}$$

$$5$$

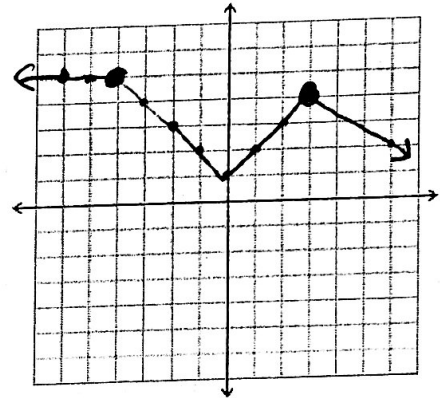
x	y
-6	5
-5	5
-4	5

$$1 + |x|$$

x	y
-4	5
-3	4
-2	3
-1	2
0	1
1	2
2	3
3	4

$$-\frac{2}{3}x + 6$$

x	y
3	4
6	2



Example 6

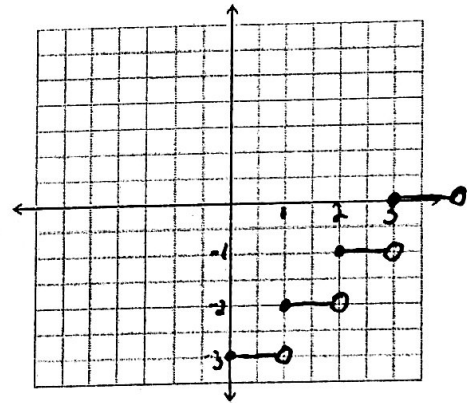
Graph and evaluate:

$$f(x) = \llbracket x \rrbracket - 3$$

$$f(0) = \underline{-3}$$

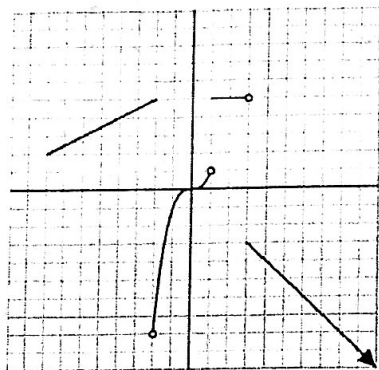
$$f(1) = \underline{-2}$$

$$f(6.1) = \underline{3}$$



Example 7

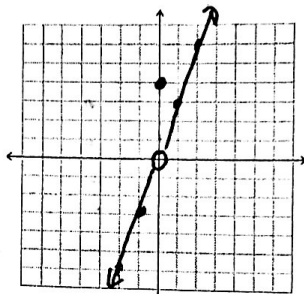
Write an equation for the piecewise function graphed.



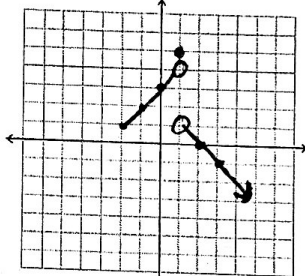
$$f(x) = \begin{cases} \frac{1}{2}x + 6, & -3 \leq x < -1 \\ x^3, & -1 < x < 1 \\ 5, & 1 \leq x < 3 \\ -x, & x \geq 3 \end{cases}$$

Part 1: Graph each piecewise function:

1. $f(x) = \begin{cases} 3x & \text{if } x \neq 0 \\ 4 & \text{if } x = 0 \end{cases}$

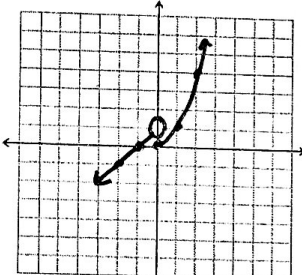


2. $f(x) = \begin{cases} x+3 & \text{if } -2 \leq x < 1 \\ 5 & \text{if } x = 1 \\ -x+2 & \text{if } x > 1 \end{cases}$



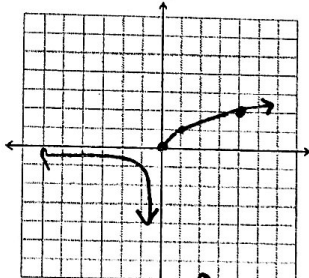
$$\begin{array}{r|l} -2 & 1 \\ -1 & 2 \\ 0 & 3 \\ 1 & 4 \\ \hline & 0 \end{array} \quad \begin{array}{r} \sqrt{5} \\ 1 \end{array} \quad \begin{array}{r|l} 1 & 1 \\ 2 & 0 \\ 3 & -1 \\ \hline & \end{array}$$

3. $f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$



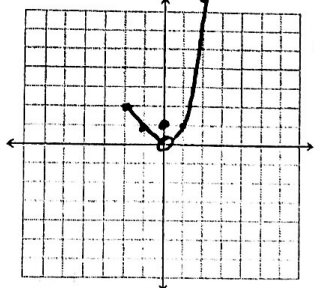
$$\begin{array}{r|l} 1+x & \\ x & y \\ -2 & -1 \\ -1 & 0 \\ 0 & 1 \\ \hline & 0 \end{array} \quad \begin{array}{r|l} x^2 & \\ x & y \\ 0 & 0 \\ 1 & 1 \\ 2 & 4 \\ \hline & \end{array}$$

4. $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$



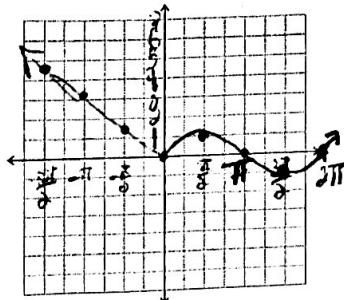
$$\begin{array}{r|l} \frac{1}{x} & \\ x & y \\ -3 & -\frac{1}{3} \\ -2 & -\frac{1}{2} \\ -1 & -1 \\ 0 & 0 \\ \hline & \end{array} \quad \begin{array}{r|l} \sqrt{x} & \\ x & y \\ 0 & 0 \\ 1 & 1 \\ 4 & 2 \\ \hline & \end{array}$$

5. $f(x) = \begin{cases} |x| & \text{if } -2 \leq x < 0 \\ 1 & \text{if } x = 0 \\ x^3 & \text{if } x > 0 \end{cases}$



$$\begin{array}{r|l} |x| & \\ x & y \\ -2 & 2 \\ -1 & 1 \\ 0 & 0 \\ \hline & \end{array} \quad \begin{array}{r|l} 1 & \\ x & y \\ 0 & 1 \\ \hline & \end{array} \quad \begin{array}{r|l} x^3 & \\ x & y \\ 0 & 0 \\ 1 & 1 \\ 2 & 8 \\ \hline & \end{array}$$

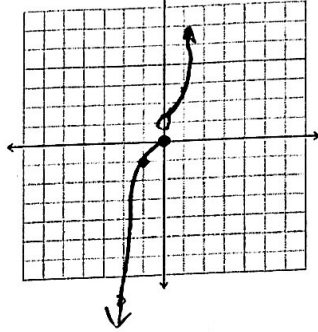
6.
$$h(x) = \begin{cases} |x| & \text{if } x < 0 \\ \sin x & \text{if } x \geq 0 \end{cases}$$



$ x $	y
0	0
$\frac{\pi}{2}$	1.57
π	3.14
$\frac{3\pi}{2}$	4.71

$\sin x$	x/y
0	0
1	$\frac{\pi}{2} \approx 1.57$
0	$\pi \approx 3.14$
-1	$\frac{3\pi}{2} \approx 4.71$
0	$2\pi \approx 6.28$

7.
$$g(x) = \begin{cases} x^3 & \text{if } x \leq 0 \\ e^x & \text{if } x > 0 \end{cases}$$



x^3	y
-8	-2
-1	-1
0	0

e^x	x/y
1	0
2.7	1

Part 2: Multiple-choice practice questions:

1. Multiple Choice Which function has range [all real numbers]?

- (a) $f(x) = 4 + \ln x$
- (b) $f(x) = 3 - 1/x$
- (c) $f(x) = 5/(1 + e^{-x})$
- (d) $f(x) = \text{int}(x - 2)$
- (e) $f(x) = 4 \cos x$

2. Multiple Choice Which function is bounded both above and below?

- (a) $f(x) = x^2 - 4$
- (b) $f(x) = (x - 3)^3$
- (c) $f(x) = 3e^x$
- (d) $f(x) = 3 + 1/(1 + e^{-x})$
- (e) $f(x) = 4 - |x|$

3. Multiple Choice Which of the following is the same as the restricted-domain function $f(x) = \text{int}(x), 0 \leq x < 2$?

(a) $f(x) = \begin{cases} 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x = 1 \\ 2 & \text{if } 1 < x < 2 \end{cases}$

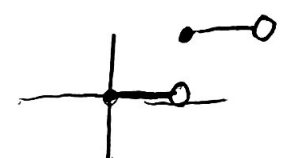
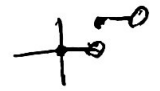
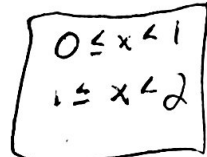
(b) $f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } 0 < x \leq 1 \\ 2 & \text{if } 1 < x < 2 \end{cases}$

(c) $f(x) = \begin{cases} 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x < 2 \end{cases}$

(d) $f(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ 2 & \text{if } 1 \leq x < 2 \end{cases}$

(e) $f(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ 1 + x & \text{if } 1 \leq x < 2 \end{cases}$

$[x]$ $0 \leq x < 2$



$$f(x) = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$$