

Steps for Adding and Subtracting Rational Expressions:

- 1) Factor, if necessary.
- 2) Divide common factors, if possible.
- 3) Look at the denominator:
  - If the denominators are the same, add/subtract the numerators and place the result over the common denominator.
  - If the denominators are different, find the LCD. Change the expressions according to the LCD and add/subtract the numerators. Place the result over the common denominator.
- 4) Reduce, if possible.
- 5) Leave the denominators in factored form.
- 6) State any restrictions on the variable.

**Example 1** Perform the indicated operation. Simplify and state any restrictions on the variable.

A.  $\frac{5}{2y} + \frac{7}{2y} : \frac{12}{2y} ; \boxed{\frac{6}{y}} \quad y \neq 0$

B.  $\frac{2x-1}{2x^2+3x-2} + \frac{2x+5}{x^2+5x+6}$       ②  $\frac{1}{x+2} \cdot \frac{x+3}{x+3} + \frac{2x+5}{(x+3)(x+2)}$       ④  $\boxed{\frac{3x+8}{(x+3)(x+2)}} \quad \begin{matrix} x \neq -3 \\ x \neq -2 \\ x \neq \frac{1}{2} \end{matrix}$

①  $\frac{2x-1}{(2x-1)(x+2)} + \frac{2x+5}{(x+3)(x+2)}$       ③  $\frac{x+3}{(x+2)(x+3)} + \frac{2x+5}{(x+3)(x+2)}$

C.  $\frac{3}{3x+9} - \frac{16x}{4x-12}$       ③  $\frac{1}{x+3} \cdot \frac{x-3}{x-3} - \frac{4x}{x-3} \cdot \frac{x+3}{x+3}$       ⑤  $\boxed{\frac{-4x^2-11x-3}{(x+3)(x-3)}} \quad \begin{matrix} x \neq 3 \\ x \neq -3 \end{matrix}$

①  $\frac{3}{3(x+3)} - \frac{4x}{4(x-3)}$       ④  $\frac{x-3}{(x+3)(x-3)} - \frac{4x^2+12x}{(x+3)(x-3)}$       ⑥  $\boxed{\frac{-(4x^2+11x+3)}{(x+3)(x-3)}}$

②  $\frac{1}{x+3} - \frac{4x}{x-3}$

D.  $\frac{x}{2x^2+7x-15} + \frac{x+1}{2x^2-11x+12}$

①  $\frac{x}{(2x-3)(x+5)} + \frac{x+1}{(2x-3)(x-4)}$

②  $\frac{x}{(2x-3)(x+5)} \cdot \frac{x-4}{x-4} + \frac{x+1}{(2x-3)(x-4)} \cdot \frac{x+5}{x+5}$

③  $\frac{x^2-4x+x^2+5x+1x+5}{(2x-3)(x+5)(x-4)}$

④  $\boxed{\frac{2x^2+2x+5}{(2x-3)(x+5)(x-4)}} \quad \begin{matrix} x \neq \frac{3}{2} \\ x \neq -5 \end{matrix}$

E.  $\frac{2}{x^2-2x} + \frac{1}{x} - \frac{3}{x^2-4}$

①  $\frac{2}{x(x-2)} \cdot \frac{x+2}{x+2} + \frac{1}{x} \cdot \frac{x^2-4}{(x-2)(x-2)} + \frac{-3}{(x+2)(x-2)} \cdot \frac{x}{x}$

②  $\frac{2x+4+x^2-4-3x}{x(x+2)(x-2)}$       ④  $\frac{x(x-1)}{x(x+2)(x-2)}$

③  $\frac{x^2-x}{x(x+2)(x-2)}$       ⑤  $\boxed{\frac{x-1}{(x+2)(x-2)}} \quad \begin{matrix} x \neq 0 \\ x \neq 2 \\ x \neq -2 \end{matrix}$

F.  $\frac{x+2}{4x} + \frac{x}{3x^2+9x}$

(2)  $\frac{3(x^2+5x+6) + 4x}{12x(x+3)}$

(1)  $\frac{x+2}{4x} \cdot \frac{x+3}{x+3} \cdot \frac{3}{3} + \frac{x}{3x(x+3)} \cdot \frac{4}{4}$

(3)  $\frac{3x^2+19x+18}{12x(x+3)} \quad \begin{matrix} x \neq 0 \\ x \neq -3 \end{matrix}$

G.  $\frac{2x}{x^2-1} - \frac{x+2}{x^2+2x+1}$

(2)  $\frac{2x^2+2x - (x^2+2x-x-2)}{(x+1)^2(x-1)}$

(1)  $\frac{x+1}{x+1} \cdot \frac{2x}{(x+1)(x-1)} - \frac{(x+2)}{(x+1)(x+1)} \cdot \frac{x-1}{x-1}$

(5)  $\frac{x^2+x+2}{(x+1)^2(x-1)} \quad \begin{matrix} x \neq 1 \\ x \neq -1 \end{matrix}$

H.  $\frac{3}{x} - \frac{5x}{x^3+1} + \frac{1}{x^2-1}$

$\frac{3}{x} \cdot \frac{(x^3+1)(x-1)}{(x^3+1)(x-1)} - \frac{5x}{(x+1)(x^2-x+1)} \cdot \frac{x(x-1)}{x(x-1)} + \frac{1}{(x+1)(x-1)} \cdot \frac{x(x^2-x+1)}{x(x^2-x+1)}$

### Simplifying Complex Fractions

One way to work with compound fractions is to clearly identify a "top" and "bottom" and simplify what is on the "top" as if it were its own problem. Meanwhile you will do the same thing with the "bottom." After you have finished whatever must be done on the top & bottom, then you multiply by the reciprocal.

**Example 2** Simplify and state any restrictions on the variable.

(1)  $\frac{\frac{x+2}{x+2} \cdot 3 - \frac{7}{x+2}}{\frac{x-3}{x-3} \cdot 1 - \frac{1}{x-3}}$   
A.

(2)  $\frac{3x+6-7}{x+2} \cdot \frac{x-3}{x-3}$

(3)  $\frac{3x-1}{x+2} \cdot \frac{x-3}{x-4}$

(4)  $\frac{(3x-1)(x-3)}{(x+2)(x-4)}$   
 $x \neq -2, x \neq 4, x \neq 3$

(1)  $\frac{\frac{3}{x+1}}{\frac{3}{x+1} + \frac{1}{x}} \cdot \frac{x+1}{x+1}$   
B.

(2)  $\frac{\frac{3}{x+1}}{3x-x-1} \cdot \frac{x(x+1)}{x(x+1)}$

(3)  $\frac{3}{x+1} \cdot \frac{x(x+1)}{2x-1}$

(4)  $\frac{3x}{2x-1} \quad \begin{matrix} x \neq \frac{1}{2} \\ x \neq -1 \\ x \neq 0 \end{matrix}$

(1)  $\frac{\frac{1}{a^2} \cdot \frac{1}{b^2} \cdot \frac{a^2}{a^2}}{\frac{1}{a} \cdot \frac{1}{b} \cdot \frac{c}{c}}$   
C.

(2)  $\frac{b^2-a^2}{a^2b^2} \cdot \frac{ab}{ab}$

(3)  $\frac{(b-a)(b+a)}{a^2b^2} \cdot \frac{ab}{b-a}$

(4)  $\frac{b+a}{ab} \quad \begin{matrix} a \neq 0 \\ b \neq 0 \\ b \neq a \end{matrix}$

(1)  $\frac{\frac{x-2}{3x+1}}{\frac{1}{x} \cdot \frac{2}{3x+1}} \cdot \frac{x}{x}$   
D.

(2)  $\frac{\frac{x-2}{3x+1}}{3x+1-2x} \cdot \frac{x(3x+1)}{x(3x+1)}$

(3)  $\frac{x-2}{3x+1} \cdot \frac{3x^2+x}{x+1}$

(5)  $\frac{x(x-2)}{(x+1)} \quad \begin{matrix} x \neq -\frac{1}{3} \\ x \neq 0 \\ x \neq -1 \end{matrix}$

(4)  $\frac{x-2}{3x+1} \cdot \frac{x(3x+1)}{x+1}$

Precalculus Unit 3 Homework—Operations with Rational Expressions  
Day 2

Simplify each expression.

$$1) \frac{5}{6ab} - \frac{7}{8a} ; \frac{-21b+20}{24ab}$$

$$2) 3x - 5 - \frac{x-8}{x+4} ; \frac{3(x^2+2x-4)}{x+4}$$

$$3) \frac{4}{a-3} + \frac{9}{a-5} ; \frac{13a-47}{(a-5)(a-3)}$$

$$4) \frac{16}{x^2-16} + \frac{2}{x+4} ; \frac{2}{x-4}$$

$$5) \frac{5}{2x-12} - \frac{20}{x^2-4x-12} ; \frac{5}{2(x+2)}$$

$$6) \frac{2-5m}{m-9} + \frac{4m-5}{9-m} ; \frac{-9m+7}{m-9}$$

$$7) \frac{2p-3}{p^2-5p+6} - \frac{5}{p^2-9} ; \frac{2p^2-2p+1}{(p-3)(p+3)(p-2)}$$

$$8) \frac{1}{5n} - \frac{3}{4} + \frac{7}{10n} ; \frac{-3(5n+6)}{20n}$$

$$9) \frac{\frac{r+6}{r} - \frac{1}{r+2}}{\frac{r^2+4r+3}{r^2+r}} ; \frac{r+4}{r+2}$$

$$10) \frac{n+5 - \frac{12}{n+1}}{\frac{n+9}{n+1} - \frac{5}{n}} ; \frac{n(n+7)}{n+5}$$

$$11) \frac{\frac{2}{x-y} + \frac{1}{x+y}}{\frac{1}{x-y}} ; \frac{3x+y}{x+y}$$

$$12) \frac{\frac{x-5x}{x+2}}{\frac{x-3}{x}} ; \frac{x^2}{x+2}$$

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$$\textcircled{1} \frac{4}{7} \cdot \frac{5}{6ab} - \frac{7}{8a} \cdot \frac{3b}{3b}$$

$$\frac{20 - 21b}{24ab}$$

$$\boxed{\frac{-21b + 20}{24ab}} \quad \begin{matrix} a \neq 0 \\ b \neq 0 \end{matrix}$$

$$\textcircled{2} \frac{x+4}{x+4} \cdot 3x - 5 \cdot \frac{x+4}{x+4} \cdot \frac{x-8}{x+4}$$

$$\frac{3x^2 + 12x - 5x - 20}{x+4} = \frac{-x+8}{x+4}$$

$$\frac{3x^2 + 6x - 12}{x+4}$$

$$\boxed{\frac{3(x^2 + 2x - 4)}{x+4}}$$

$$\textcircled{3} \frac{4}{a-3} \cdot \frac{a-5}{a-5} + \frac{9}{a-5} \cdot \frac{a-3}{a-3}$$

$$\frac{4a - 20 + 9a - 27}{(a-5)(a-3)}$$

$$\boxed{\frac{13a - 47}{(a-5)(a-3)}}$$

$$\textcircled{4} \frac{16}{(x+4)(x-4)} + \frac{2}{x+4} \cdot \frac{x-4}{x-4}$$

$$\frac{16 + 2x - 8}{(x+4)(x-4)}$$

$$\frac{2x + 8}{x+4 \cdot x-4}$$

$$\frac{2(x+4)}{(x+4)(x-4)}$$

$$\boxed{\frac{2}{x-4}}$$

$$\textcircled{5} \frac{5}{2(x-6)} \cdot \frac{x+2}{x+2} + \frac{-20}{(x-6)(x+2)} \cdot \frac{2}{2}$$

$$\frac{5x + 10 - 40}{2(x+2)(x-6)}$$

$$\frac{5x - 30}{2(x+2)(x-6)}$$

$$\frac{5(x-6)}{2(x+2)(x-6)}$$

$$\boxed{\frac{5}{2(x+2)}}$$

$$\textcircled{6} \frac{2-5m}{m-9} + \frac{4m-5}{-(m-9)}$$

$$\frac{2-5m-4m+5}{m-9}$$

$$\boxed{\frac{-9m+7}{m-9}}$$

$$\textcircled{7} \frac{2p-3}{(p-3)(p-2)} \cdot \frac{p+3}{p+3} + \frac{-5}{(p+3)(p-3)} \cdot \frac{p-2}{p-2}$$

$$\frac{2p^2 + 6p - 3p - 9 - 5p + 10}{(p+3)(p-3)(p-2)}$$

$$\frac{2p^2 - 2p + 1}{(p+3)(p-3)(p-2)}$$

$$\textcircled{8} \frac{1}{5n} \cdot \frac{4}{4} - \frac{3}{4} \cdot \frac{5n}{5n} + \frac{7}{10n} \cdot \frac{2}{2}$$

$$\frac{4 - 15n + 14}{20n}$$

$$\frac{-15n + 18}{20n}$$

$$\frac{-3(5n-6)}{20n}$$

$$\textcircled{9} \frac{r+3}{r+2} \cdot \frac{r+6}{r} + \frac{-1}{r+2} \cdot \frac{r}{r}$$

$$\frac{r^2 + 8r + 12 - r}{r(r+2)}$$

$$\frac{(r+3)(r+1)}{r(r+1)}$$

$$\frac{(r+3)(r+1)}{r(r+2)} \cdot \frac{r}{(r+3)(r+1)}$$

$$\boxed{\frac{r+4}{r+2}}$$

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$$(10) \frac{\frac{n}{n+1}(n+5)}{n} - \frac{12}{n+1}$$

$$\frac{n}{n} \cdot \frac{n+5}{n+1} - \frac{5}{n} \cdot \frac{n+1}{n+1}$$

$$\frac{n^2 + 5n - 5}{n+1}$$

$$\frac{n^2 + 9n - 5n - 5}{n(n+1)}$$

$$\frac{n^2 + 6n - 7}{n+1}$$

$$\frac{n^2 + 4n - 5}{n(n+1)}$$

$$\frac{(n+7)(n-1)}{n+1} \cdot \frac{n(n+1)}{(n+5)(n-1)}$$

$$\boxed{\frac{n(n+7)}{n+5}}$$

$$(11) \frac{\frac{2}{x-y} \cdot \frac{x+y}{x+y} + \frac{1}{x+y} \cdot \frac{x-y}{x-y}}{\frac{1}{x-y}}$$

$$\frac{1}{x-y}$$

$$\frac{2x + 2y + x - y}{(x-y)(x+y)}$$

$$\frac{1}{x-y}$$

$$\frac{3x + y}{(x-y)(x+y)} \cdot \frac{x-y}{1}$$

$$\boxed{\frac{3x + y}{x + y}}$$

$$(12) \frac{x \cdot \frac{x+2}{x+2} - 5x}{x-3}$$

$$\frac{x^2 + 2x - 5x}{x+2}$$

$$\frac{x-3}{x}$$

$$\frac{x^2 - 3x}{x+2}$$

$$\frac{x-3}{x}$$

$$\frac{x(x-3)}{x+2} \cdot \frac{x}{x-3}$$

$$\boxed{\frac{x^2}{x+2}}$$