



A "rational expression" is the quotient of two polynomials.

A rational expression is in *simplest form* when the numerator and denominator have no common factors other than 1.

To simplify rational expressions ...

- 1) Factor the numerator and denominator completely.
- 2) Divide common factors.
- 3) State restrictions on the variable--some may no longer be visible in the final answer.

Example 1 Simplify. State any restrictions on the variable.

A. $\frac{x^2+6x+5}{x^2+3x-10}$; ~~$\frac{(x+2)(x+3)}{(x+2)(x-5)}$~~ $\frac{(x+5)(x+1)}{(x+5)(x-2)} = \frac{x+1}{x-2}$ $x \neq 2$
 $x \neq -5$

B. $\frac{6x^2-7x-3}{8x^2-2x-15}$; ~~$\frac{(3x+1)(2x-3)}{(2x-3)(4x+5)}$~~ $\frac{3x+1}{4x+5}$ $x \neq -\frac{5}{4}$
 $x \neq \frac{3}{2}$

C. $\frac{5x}{25x^2+10x}$; $\frac{5x}{5x(5x+2)}$; $\frac{1}{5x+2}$ $x \neq 0$
 $x \neq -\frac{2}{5}$

D. $\frac{2x^2+5x-12}{15-10x}$; ~~$\frac{(2x+3)(x+4)}{-5(2x-3)}$~~ ; $\frac{x+4}{-5}$ $x \neq \frac{3}{2}$

E. $\frac{y^2-36}{2y^2-7y-30}$; ~~$\frac{(y+6)(y-6)}{(2y+5)(y-6)}$~~ ; $\frac{y+6}{2y+5}$ $y \neq -\frac{5}{2}$
 $y \neq 6$

F. $\frac{x^2-3x}{x^2-9}$; ~~$\frac{x(x-3)}{(x+3)(x-3)}$~~ ; $\frac{x}{x+3}$ $x \neq -3$
 $x \neq 3$

Things to keep in mind when *multiplying or dividing* rational expressions...

- Only divide out FACTORS (expressions being multiplied)
- Pay close attention to location (top versus bottom)
- To divide by a fraction, multiply by the reciprocal.

Example 2 Perform the indicated operation. Be sure to simplify and state any restrictions on the variable.

A. $\frac{x^2 - 6x - 7}{x^2 + 5x + 4} \times \frac{2x^2 + 9x + 4}{3x^2 - 23x + 14} = \frac{(x-7)(x+1)}{(x+4)(x+1)} \cdot \frac{(2x+1)(x-4)}{(3x-2)(x-7)} = \frac{2x+1}{3x-2}$
 Restrictions: $x \neq -4, x \neq -1, x \neq 2/3, x \neq 7$

B. $\frac{2x^2 + 11x - 21}{x^3 + 2x^2 + 4x} \cdot \frac{x^3 - 8}{x^2 + 5x - 14} = \frac{(2x-3)(x+7)}{x(x^2+2x+4)} \cdot \frac{(x-2)(x^2+2x+4)}{(x+7)(x-2)} = \frac{2x-3}{x}$
 Restrictions: $x \neq -7, x \neq 0, x \neq 2$

C. $\frac{4c^2 + 8cd + 4d^2}{8} \cdot \frac{3c - 3d}{c^2 - d^2} = \frac{4(c^2 + 2cd + d^2)}{2 \cdot 4} \cdot \frac{3(c-d)}{(c-d)(c+d)} = \frac{3(c-d)}{2(c+d)}$
 Restrictions: $c \neq d, c \neq -d$

D. $\frac{x^2 - 9}{x^2 + 5x + 4} \div \frac{x^2 + 4x + 3}{x^2 + 7x + 12} = \frac{(x+3)(x-3)}{(x+4)(x+1)} \cdot \frac{(x+4)(x+3)}{(x+3)(x+1)} = \frac{(x-3)(x+3)}{(x+1)^2}$
 Restrictions: $x \neq -4, x \neq -1, x \neq -3$

E. $\frac{3t^2 + 10t - 8}{4t^2 - 12t + 9} \div \frac{6t^2 - 13t + 6}{4t^2 - 9} = \frac{(3t-2)(t+4)}{(2t-3)(2t-3)} \cdot \frac{(2t-3)(2t+3)}{(3t-2)(2t-3)} = \frac{(t+4)(2t+3)}{(2t-3)^2}$
 Restrictions: $t \neq 2/3, t \neq 3/2, t \neq -3/2$

F. $\frac{x^3 + 1}{x^2 - x - 2} \div \frac{x^2 - x + 1}{x^2 - 4x + 4} = \frac{(x+1)(x^2 - x + 1)}{(x-2)(x+1)} \cdot \frac{(x-2)(x-2)}{x^2 - x + 1} = x - 2$
 Restrictions: $x \neq 2, x \neq -1$

Precalculus Unit 3 Homework—Operations with Rational Expressions
Day 1

In exercises 1-3, find the missing numerator or denominator so that the two rational expressions are equal.

1. $\frac{2}{3x} = \frac{?}{12x^3}$ $8x^2$ 2. $\frac{x-4}{x} = \frac{x^2-4x}{?}$ x^2 3. $\frac{x+3}{x-2} = \frac{?}{x^2+2x-8}$ $x^2+7x+12$

In exercises 4-8, write the expression in reduced form.

4. $\frac{18x^3}{15x}$ $\frac{6x^2}{5}$ 5. $\frac{x^3}{x^2-2x}$ $\frac{x^2}{x-2}$ 6. $\frac{z^2-3z}{9-z^2}$ $\frac{z}{-(z+3)}$

7. $\frac{y^2-y-30}{y^2-3y-18}$ $\frac{y+5}{y+3}$ 8. $\frac{8z^3-1}{2z^2+5z-3}$ $\frac{(2z-1)(4z^2+2z+1)}{(2z-1)(z+3)}$: $\frac{4z^2+2z+1}{z+3}$

In exercises 9-18, simplify.

9. $\frac{3}{x-1} \cdot \frac{x^2-1}{9}$: $\frac{x+1}{3}$ 10. $\frac{x+3}{x-1} \cdot \frac{1-x}{x^2-9}$ $\frac{-1}{x-3}$

11. $\frac{x^3-1}{2x^2} \cdot \frac{4x}{x^2+x+1}$ $\frac{2(x-1)}{x}$ 12. $\frac{2y^2+9y-5}{y^2-25} \cdot \frac{y-5}{2y^2-y}$: $\frac{(2y-1)(y+5)}{y+5} \cdot \frac{1}{2(2y-1)}$
: $\frac{1}{y}$

13. $\frac{y^3+2y^2+4y}{y^3+2y^2} \cdot \frac{y^2-4}{y^3-8}$: $\frac{1}{y}$

14. $\frac{1}{2x} \div \frac{1}{4}$: $\frac{2}{x}$

15. $\frac{x^2-3x}{14y} \div \frac{2xy}{3y^2}$: $\frac{3(x-3)}{28}$ 16. $\frac{2x^2y}{(x-3)^2} \div \frac{8xy}{x-3}$: $\frac{x}{4(x-3)}$

17. $\frac{7x-7y}{4y} \div \frac{14x-14y}{3y}$: $\frac{3}{8}$ 18. $\frac{x^2-y^2}{2xy} \div \frac{2xy}{y^2-x^2}$: $\frac{(x+y)(x-y)}{2xy} \cdot \frac{2xy}{-(x-y)(x+y)}$
: $-2x$ $\begin{matrix} x \neq 0 \\ y \neq 0 \\ x \neq y \\ x \neq -y \end{matrix}$