Pre- Calculus

Key

REVIEW FOR MIDTERM

Chapter 1 – Functions and their properties:

1) State the domain of each of the functions below:

a) $f(x) = \sqrt{3x-4}$

$x \geq 4/3$

$\left[\frac{4}{3}, \infty\right)$

b) $f(x) = \frac{\sqrt{7-x}}{x-4}$

$7-x \geq 0$ $\quad x \neq 4$

$7 \geq x$

$x \leq 7$

$(-\infty, 4) \cup (4, 7]$

c) $f(x) = \ln(3x-7)$

$3x - 7 > 0$

$x > \frac{7}{3}$

$\left(\frac{7}{3}, \infty\right)$

d) $f(x) = \frac{\sqrt{3-x}}{2x^2+x-3}$

$3-x \geq 0$ $\quad (2x+3)(x-1) \neq 0$

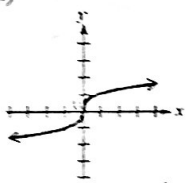
$3 \geq x$ $\quad x \neq -\frac{3}{2}$ $\quad x \neq 1$

$x \leq 3$

$\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, 1\right) \cup (1, 3]$

2) State the interval over which each of the following are increasing/decreasing:

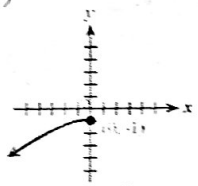
a)



I: $(-\infty, \infty)$

D: None

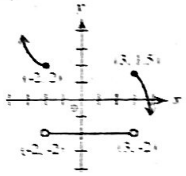
b)



I: $(-\infty, 0]$

D: None

c)

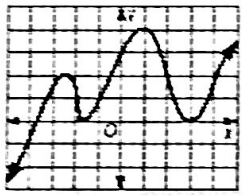


I: none

D: $(-\infty, -2] \cup [3, \infty)$

3) Which of the following are functions? A + D

a)



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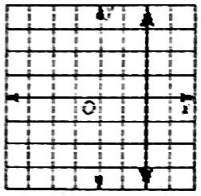
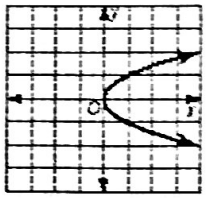
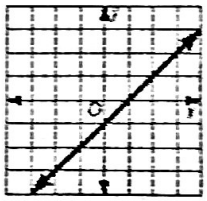


c)



d)



4) Determine whether each of the following are even, odd, or neither:

a) $f(x) = \frac{3x}{x^2-4}$

$f(1) = \frac{3}{1-4} = \frac{3}{-3} = -1$ $\quad f(-1) = \frac{-3}{1-4} = \frac{-3}{-3} = 1$

Odd

b) $y = 5|x| - 7x^4 + 11$

$f(1) = 5 - 7 + 11 = 9$ $\quad f(-1) = 5 - 7 + 11 = 9$

Even

c) $f(x) = 5x^7 - 4x^3 - x$

$f(1) = 5 - 4 - 1 = 0$ $\quad f(-1) = -5 + 4 + 1 = 0$

$f(x) = 5x^7 - 4x^3 - x$ $\quad f(-x) = -5x^7 + 4x^3 + x$

odd

d) $y = \sqrt{2x-9}$

$f(1) = \sqrt{2-9}$ $\quad f(-1) = \sqrt{-2-9}$

$f(1) = \sqrt{-7}$ $\quad f(-1) = \sqrt{-11}$

Neither

5) Determine the functions implicitly defined below:

a) $x^2 + y^2 = 9$

$y^2 = -x^2 + 9$

$y = \pm\sqrt{-x^2+9}$

b) $y^2 - x = 2$

$y^2 = x + 2$

$y = \pm\sqrt{x+2}$

c) $9x^2 + 16y^2 = 100 - 24xy$

$9x^2 + 24xy + 16y^2 = 100$

$\sqrt{(3x+4y)^2} = \sqrt{100}$

$3x + 4y = \pm 10$

$\frac{4y}{4} = \frac{-3x \pm 10}{4}$

$y = -\frac{3}{4}x \pm 10$

or

$y = -\frac{3}{4}x + 10$

$y = -\frac{3}{4}x - 10$

6) Use the given functions to find the compositions and their domains

$f(x) = x^2 - 3x$ $\quad g(x) = \frac{3}{x+1}$ $\quad h(x) = 2\sqrt{1-x}$ $\quad k(x) = \sqrt{x+3}$

a) $(f \circ k)(x)$

$(\sqrt{x+3})^2 - 3(\sqrt{x+3})$

$x + 3 - 3\sqrt{x+3}$

D: $[-3, \infty)$

b) $(h \circ g)(x)$

$2\sqrt{1 - \frac{3}{x+1}}$

$2\sqrt{\frac{x+1-3}{x+1}}$

$2\sqrt{\frac{x-2}{x+1}}$

$x \neq -1$

$x \geq -1$

$x \geq 2$

D: $[2, \infty)$

c) $g(f(x))$

$\frac{3}{x^2 - 3x + 1}$

$x = \frac{--3 \pm \sqrt{9 - 4(1)(1)}}{2(1)}$

$x \neq \frac{3 \pm \sqrt{5}}{2}$

D: $\left(-\infty, \frac{3-\sqrt{5}}{2}\right) \cup \left(\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right) \cup \left(\frac{3+\sqrt{5}}{2}, \infty\right)$

d) $f(h(x))$

$(2\sqrt{1-x})^2 - 3(2\sqrt{1-x})$

$4(1-x) - 3(2\sqrt{1-x})$

$4 - 4x - 6\sqrt{1-x}$

D: $(-\infty, 1]$

7) Each function below is a transformation of one of the 12 basic functions. Describe the transformations.

a) $f(x) = e^{-2x+8} - 7$

$f(x) = e^{-2(x-4)} - 7$

reflect over y

horiz shrink by $\frac{1}{2}$

$\rightarrow 4$ $\downarrow 7$

b) $g(x) = \frac{1}{2 + 2e^{-x+5}}$

$g(x) = \frac{1}{2(1 + e^{-(x-5)})}$

vertical shrink by $\frac{1}{2}$

$\rightarrow 5$

c) $f(x) = 9\cos(2x) - 11$

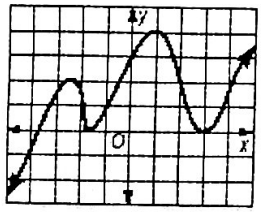
vert. stretch by 9

horiz shrink by $\frac{1}{2}$

$\downarrow 11$

8) Which of the following have inverses that are functions? B, C, & D

a)



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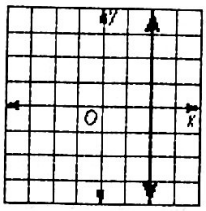
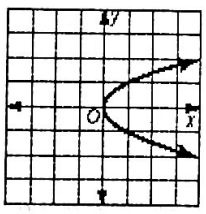
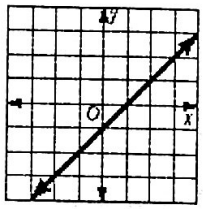


c)



d)



Chapter 2A – Polynomial & Power Functions:

1) Write the following quadratic functions in vertex form then state the vertex:

a) $f(x) = x^2 - 8x + 11$

$y - 11 + 16 = x^2 - 8x + 16$

$y = (x-4)^2 - 5$

b) $y = 3x^2 + 6x + 5$

$\frac{y-5}{3} + 1 = x^2 + 2x + 1$

$\frac{y-5}{3} = (x+1)^2 - 1$

$y - 5 = 3(x+1)^2 - 3$

$y = 3(x+1)^2 + 2$

2) Find the zeros of the polynomial functions below:

a) $f(x) = x^3 - 3x^2 - 25x + 75$

$x^2(x-3) - 25(x-3)$

$y = (x+5)(x-5)(x-3)$

$x = -5 \quad x = 5 \quad x = 3$

b) $f(x) = x^4 - 11x^2 + 28$

$f(x) = (x^2-7)(x^2-4)$

$x = \pm\sqrt{7} \quad x = \pm 2$

c) $f(x) = x^3 - 2x^2 - 5x + 6$

	1	-2	-5	6
		1	-6	
	1	-1	-6	0

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3 \quad x = -2 \quad x = 1$

3) Use long division or synthetic (you should look in your notes to determine which to use when) to find:

a) $\dfrac{3x^2 - 7x + 2}{x+5}$

-5	3	-7	2
		-15	110
	3	-22	112

$3x - 22 + \dfrac{112}{x+5}$

b) $\dfrac{2x^4 - x^3 - 2}{2x^2 + x + 1}$

$x^2 - x + \dfrac{x-2}{2x^2+x+1}$

$2x^2+x+1 \;\overline{)\; 2x^4 - x^3 + 0x^2 + 0x - 2}$

$x^2 - x + \dfrac{x-2}{2x^2+x+1}$

$-(2x^4 + x^3 + x^2)$

$-2x^3 - x^2 + 0x$

$-(-2x^3 - x^2 - x)$

$x - 2$

4) State the complex conjugate of each of the following and then find their product:

a) $7 - 3i$

$7 + 3i$

$49 - 9i^2 \rightarrow 49 + 9$

$49 - 9(-1)$

58

b) $-5 + 8i$

$-5 + \bar{8}i$

$25 - 64i^2$

89

5) Find all the complex zeros of the following polynomial functions:

a) $y = x^3 - 2x^2 + 4x - 8$

$x^2(x-2) + 4(x-2)$

$x = \pm 2i \quad x = 2$

b) $f(x) = x^2 - 6x + 25$

$x = \dfrac{-6 \pm \sqrt{36 - 4(1)(25)}}{2(1)}$

$x = \dfrac{+6 \pm 8i}{2}$

$x = 3 \pm 4i$

c) $y = 7x^2 + 42$

$0 = 7x^2 + 42$

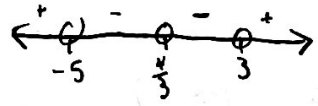
$\dfrac{-42}{7} = \dfrac{7x^2}{7}$

$\sqrt{x^2} = \sqrt{-6}$

$x = \pm i\sqrt{6}$

6) Solve the following non-linear inequalities:

a) $(3x-4)^2(x+5)(x-3) > 0$ $\quad \frac{4}{3}, -5, 3$



$(-\infty, -5) \cup (3, \infty)$

b) $x^3 + 9x^2 + 11x \leq 21$

$x^3 + 9x^2 + 11x - 21 \leq 0$

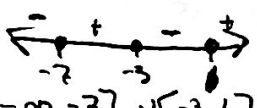
1	1	9	11	-21
		1	10	21
	1	10	21	0

$x = 1 \quad x = -7$

$x = -3$

$x^2 + 10x + 21$

$(x-1)(x+7)(x+3) \leq 0$



$(-\infty, -7] \cup [-3, 1]$

c) $\dfrac{x-8}{x^2-4} \geq \dfrac{-1}{x+2}$

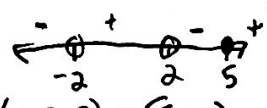
$\dfrac{x-8}{x+2\;x-2} + \dfrac{1}{x+2} \cdot \dfrac{x-2}{x-2} \geq 0$

$\dfrac{2x-10}{(x+2)(x-2)} \geq 0$

$x \neq 2$

$x \neq -2$

$x = 5$



$(-2, 2) \cup [5, \infty)$

7) Determine whether each of the following is a polynomial. If it is state the degree & leading coefficient (if not put "n/a"):

a. $f(x)=5x^{3.2}+4x-3$

This is not (is/is not) a polynomial.

Degree = ___ & L.C. = ___

b. $f(x)=7x^2-3x+8$

This is (is/is not) a polynomial.

Degree = 2 & L.C. = 7

c. $f(x)=\frac{7x+5}{3x}$

This is not (is/is not) a polynomial.

Degree = ___ & L.C. = ___

d. $f(x)=\frac{7x+5}{3}$

This is (is/is not) a polynomial.

Degree = 1 & L.C. = 7/3

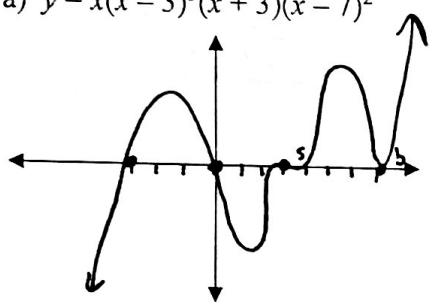
e. $f(x)=\sqrt[3]{18x^3+4x^2+10}$

This is not (is/is not) a polynomial.

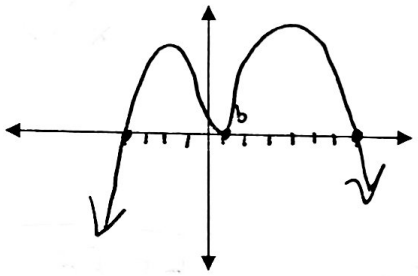
Degree = ___ & L.C. = ___

8) Sketch each of the following polynomials:

a) $y=x(x-3)^3(x+3)(x-7)^2$



b) $f(x)=(7-x)(x-1)^2(x+4)$ $\quad -(x-7)(x-1)^2(x+4)$



Chapter 2B – Rational Functions & Non-Linear Inequalities:

1) Simplify each of the following rational expressions. State any restricted values.

a) $\frac{x^2+6x+5}{x^2+3x-10}$

$\frac{(x+5)(x+1)}{(x+5)(x-2)}$

$\frac{x+1}{x-2}$ $\quad x\neq -5,\ x\neq 2$

b) $\frac{20x^3+30x^2+45x}{40x^4-135x}$

$\frac{5x(4x^2+6x+9)}{5x(8x^3-27)}$

$\frac{(2x+3)(2x+3)}{(2x-3)(4x^2+6x+9)}\cdot\frac{4x^2+6x+9}{(2x+3)(2x+3)}$

$\frac{1}{2x-3}$ $\quad x\neq 0,\ x\neq \frac{3}{2}$

c) $\frac{\frac{5}{x-2}}{\frac{1}{x-2}+\frac{2}{x+1}}\cdot\frac{\frac{x+1}{x+1}}{\frac{x-2}{x-2}}$

(1) $\frac{5}{x-2}\cdot\frac{x+1}{x+1+2x-4}$

(2) $\frac{5}{x-2}\cdot\frac{(x-2)(x+1)}{3x-3}$

(3) $\frac{5(x+1)}{3(x-1)}$ $\quad x\neq 1,\ x\neq 2,\ x\neq -1$

2) Determine each of the x- & y-intercepts and state the left and right end behavior for each of the following:

a. $f(x)=x^4-5x^2+4$

$x^2-4\ \ x^2-1$

$(x+2)(x-2)(x+1)(x-1)$

y-int: (0, 4)

Zero(s): (-2, 0) (2, 0) (1, 0) (-1, 0)

LEB: $\lim_{x\to-\infty} f(x)=\infty$

REB: $\lim_{x\to\infty} f(x)=\infty$

b. $f(x)=x^4-2x^3+8x-16$

$x^3(x-2)+8(x-2)$

$(x+2)(x^2-2x+4)(x-2)$

y-int: (0, -16)

Zero(s): (-2, 0) (2, 0)

LEB: $\lim_{x\to-\infty} f(x)=\infty$

REB: $\lim_{x\to\infty} f(x)=\infty$

c. $f(x)=81x^4-16$

$(9x^2-4)(9x^2+4)$

$(3x-2)(3x+2)(9x^2+4)$

y-int: (0, -16)

Zero(s): $(\frac{2}{3},0)$ $(-\frac{2}{3},0)$

LEB: $\lim_{x\to-\infty} f(x)=\infty$

REB: $\lim_{x\to\infty} f(x)=\infty$

3) Write the equations of all of the asymptotes of the function:

a) $y=\dfrac{5x^3-4x^2+1}{x^3-8}$ $(x-2)(x^2+2x+4)$

VA: $x=2$

HA: $y=5$

b) $y=\dfrac{5x^2-20x^2+20}{x^3-8}$ $:(x-2)(x^2+2x+4)$

VA: $x=2$

HA: $y=0$

c) $y=\dfrac{x-10}{x^2-9}$ $:x+3\ x-3$

VA: $x=3\ \ x=-3$

HA: $y=0$

4) Solve each of the following rational equations

a) $\dfrac{x-2}{x+4}+\dfrac{x+1}{x+6}=\dfrac{11x+32}{x^2+10x+24}$ $(x+6)(x+4)$

$x+6\quad x+4$

$(x-2)(x+6)+(x+1)(x+4)=11x+32$

$x^2+4x-12+x^2+5x+4=11x+32$

$2x^2-2x-40=0$

$x^2-x-20=0$

$(x-5)(x+4)=0$

$x=5$ ~~$x=-4$~~

b) $\dfrac{y+3}{y+2}=1-\dfrac{y+1}{y+2}$ $(y+2)$

$y+3=y+2-y-1$

$y+3=1$

$y=-2$

No Solution

Chapter 3 – Logistic, Exponential, & Logarithmic Functions:

1) Write each of the following in exponential form:

a) $\log 7=x$

$10^x=7$

b) $\log_4 16=2$

$4^2=16$

c) $\log_u k=z$

$u^z=K$

2) Write each of the following in logarithmic form:

a) $7^3=343$

$\log_7 343=3$

b) $11^x=3$

$\log_{11} 3=x$

c) $t^4=11$

$\log_t 11=4$

3) Evaluate each of the following logarithms:

a) $\log_8 4=$ $\frac{2}{3}$

$8^x=4$

$2^{3x}=2^2$

b) $\log_9 27^{1/3}$

$9^x=27^{\frac{1}{3}}$

$3^{2x}=(3^3)^{\frac{1}{3}}$

$x=\frac{1}{2}$

c) $\ln e^{4x}$

$4x$

d) $\log 0.001$

$\log 10^{-3}$

-3

4) Expand each of the following logarithms:

a) $\ln(7xy)$

$\ln 7+\ln x+\ln y$

b) $\ln\left(\dfrac{x(x+4)}{\sqrt{x^2+1}}\right)$

$\ln x+\ln(x+4)-\frac{1}{2}\ln(x^2+1)$

c) $\log\left(\dfrac{x^3y^6}{\sqrt{z}}\right)$

$3\log x+6\log y-\frac{1}{2}\log z$

5) Condense each of the following logarithms:

a) $\log_2 x-2\log_2 y+\dfrac{1}{3}\log_2 t$

$\log \dfrac{x\sqrt[3]{t}}{y^2}$

b) $4\ln a-\left(\ln b-\dfrac{2}{3}\ln c\right)$

$\ln \dfrac{a^4\sqrt[3]{c^2}}{b}$

c) $3(5\ln x+2\ln y)+2(\ln y-7\ln x)$

$15\ln x+6\ln y+2\ln y-14\ln x$

$\ln \dfrac{x^{15}y^6y^2}{x^{14}}$

$\ln xy^8$

6) Solve each of the following logarithmic or exponential equations:

a) $\log_3 (x^2 + 8) - \log_3 4 = 3$

$\log_3 \frac{x^2+8}{4} = 3$

$3^3 = \frac{x^2+8}{4}$

$108 = x^2 + 8$

$x^2 = 100$

$\boxed{x = \pm 10}$

b) $\ln(x + 7) + \ln(x + 3) = \ln 77$

$\ln[(x+7)(x+3)] = \ln 77$

$x^2 + 10x + 21 = 77$

$x^2 + 10x - 56 = 0$

$(x+14)(x-4) = 0$

~~$x = -14$~~ $\boxed{x = 4}$

c) $\log_4 (x^2 - 3) + \log_4 10 = 1$

$\log_4 10(x^2-3) = 1$

$4^1 = 10(x^2-3)$

$4 + 30 = x^2$

$34 = x^2$

$\boxed{x = \pm\sqrt{34}}$

d) $\left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$

$6^{-3x-2} \cdot (6^3)^{3x} = 6^{-3}$

$6^{-3x-2+9x} = 6^{-3}$

$6x - 2 = -3$

$6x = -1$

$\boxed{x = -\frac{1}{6}}$

e) $9^{-3x} \cdot 9^x = 27$

$9^{-2x} = 27$

$(3^2)^{-2x} = 3^3$

$-4x = 3$

$\boxed{x = -\frac{3}{4}}$

f) $16^{2x-3} \cdot 4^{-2x} = 2^4$

$(2^4)^{2x-3} \cdot (2^2)^{-2x} = 2^4$

$2^{8x-12-4x} = 2^4$

$4x - 12 = 4$

$4x = 16$

$\boxed{x = 4}$