Precalculus Unit 2 Notes—(review) Linear & Quadratic Polynomials

Definition of a Polynomial Function

Let n be a nonnegative integer and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ be real numbers, with $a_n \neq 0$. The function defined by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

is called a polynomial function of degree n. The number a_n, the coefficient of the variable to the highest power, is called the leading coefficient.

Example 1 Determine whether each of the following is a polynomial function.

a) $f(x) = 5x^{-1}$ No

b) $g(x) = 4x^2 + ex - 10$ yes

c) $h(x) = -3x^{\pi} + 4x^3 + 11x$ No

d) $j(x) = 2x^{\frac{1}{2}} + 10x + 6$ No

e) $k(x) = \frac{1}{4}x^4 - 9x^3 + 7$ Yes

f) $f(x) = -x^4 + 5x^3 - 8ix^2 + 7$ No

Classifying Polynomials by Degree		
Name	Degree	Example
Constant	0	-9
Linear	1	$x - 4$
Quadratic	2	$x^2 + 3x - 1$
Cubic	3	$x^3 + 2x^2 + x + 1$
Quartic	4	$2x^4 + x^3 + 3x^2 + 4x - 1$
Quintic	5	$7x^5 + x^4 - x^3 + 3x^2 + 2x - 1$

****There are 2 polynomial functions you should be very familiar with... Linear & Quadratic****

LINEAR

Slope-Intercept Form: $y = mx + b$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Standard Form: $ax + by = c$

Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

QUADRATIC

Vertex Form: $y = a(x-h)^2 + K$

Standard Form: $y = ax^2 + bx + c$

Finding the Vertex: complete the square or $x = -\frac{b}{2a}$

Completing the Square:

Ways to solve quadratics: factor, complete the square, graph, + quadratic formula

Example 2 Write a linear function that satisfies all of the following conditions:

a) passes through the points (0, 3) and (-4, -1)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{-4-0} = \frac{-4}{-4} = 1$ $\quad y - 3 = 1(x - 0)$ $\quad y = x + 3$

b) $f(4) = -2$ and $f(-4) = -4$

$m = \frac{-4 - -2}{-4 - 4} = \frac{-2}{-8} = \frac{1}{4}$ $\quad y - -2 = \frac{1}{4}(x - 4)$ $\quad y + 2 = \frac{1}{4}x - 1$ $\quad y = \frac{1}{4}x - 3$

Example 3 Write an equation for the quadratic function with the given vertex and point:

$y = a(x-h)^2 + k$ (h, k) (x, y)

a) vertex (2, 0) passing through (1, 3)

$3 = a(1-2)^2 + 0$ $\quad a = 3$ $\quad y = 3(x-2)^2$

b) vertex (-3, 4) passing through (0, 0)

$0 = a(0 - -3)^2 + 4$ $\quad -4 = 9a$ $\quad a = -\frac{4}{9}$ $\quad y = -\frac{4}{9}(x+3)^2 + 4$

To convert a quadratic from standard form to vertex form:

Step 1: Check the coefficient of the x^2 term. If 1 goto step 2
If not 1, factor out the coefficient from x^2 and x terms.

Step 2: Calculate the value of : $(b/2)^2$

Step 3: Group the x^2 and x term together, then add $(b/2)^2$ and subtract $(b/2)^2$

Step 4: Factor & Simplify

Example 4 Use completing the square to write the following equations in vertex form:

a) $y = x^2 + 6x - 11$

$y + 11 + 9 = x^2 + 6x + 9$ $\quad y + 20 = (x+3)^2$ $\quad y = (x+3)^2 - 20$ $\quad V: (-3, -20)$

b) $y = 2x^2 - 12x + 1$

① $\frac{y-1}{2} + 9 = x^2 - 6x + 9$

② $\frac{y-1}{2} + 9 = (x-3)^2$ $\quad -9$

③ $\frac{y-1}{2} = (x-3)^2 - 9$

④ $y - 1 = 2(x-3)^2 - 18$ $\quad +1$ $\quad +1$

$y = 2(x-3)^2 - 17$

c) $y = -x^2 - 3x - 5$

① $\frac{y+5}{-1} + \frac{9}{4} = x^2 + 3x + \frac{9}{4}$

② $-1\left[\frac{y+5}{-1} = (x + \frac{3}{2})^2 - \frac{9}{4}\right]$

③ $y + 5 = -1(x + \frac{3}{2})^2 + \frac{9}{4}$ $\quad -5$ $\quad -5$

④ $y = -(x + \frac{3}{2})^2 - \frac{11}{4}$

d) $y = \frac{1}{3}x^2 - 4x - 1$

① $3y = x^2 - 12x - 3$

② $3y + 3 + 36 = x^2 - 12x + 36$

③ $3y + 39 = (x-6)^2$

④ $\frac{3y}{3} = \frac{(x-6)^2 - 39}{3}$

⑤ $y = \frac{1}{3}(x-6)^2 - 13$

Determine which are polynomial functions. For those that are, state the degree and leading coefficient. For those that are not, explain why not.

1. $f(x)=3x^{-5}+17$

No

2. $f(x)=-9+2x$

yes
d: 1
LC: 2

3. $f(x)=2x^5-\frac{1}{2}x+9$

yes
d: 5
LC: 2

4. $f(x)=13$

yes
d: 0
LC: 0

5. $h(x)=\sqrt[3]{27x^3+8x^6}$

NO

6. $y(x)=4x-5x^2$

yes
d: 2
LC: -5

Write an equation for the linear function f satisfying the given conditions.

7. $f(-5)=-1$ and $f(2)=4$

$m=\frac{4-(-1)}{2-(-5)}=\frac{5}{7}$

$y-4=\frac{5}{7}(x-2)$

$y+1=\frac{5}{7}(x+5)$

$y=\frac{5}{7}x+\frac{18}{7}$

8. $f(-4)=6$ and $f(-1)=2$

$m=\frac{6-2}{-4-(-1)}=\frac{4}{-3}$

$y-2=-\frac{4}{3}(x+1)$

$y-6=-\frac{4}{3}(x+4)$

$y=-\frac{4}{3}x+\frac{2}{3}$

9. $f(0)=3$ and $f(3)=0$

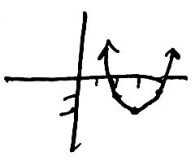
$m=\frac{0-3}{3-0}=\frac{-3}{3}=-1$

$y=-x+3$

Describe how to transform the graph of $f(x)=x^2$ into the graph of the given functions. Sketch each graph by hand.

10. $g(x)=(x-3)^2-2$

→3 ↓2



11. $h(x)=-3x^2+2$

reflect over x-axis
vert stretch by 3
↑2

Find the vertex and axis of symmetry for each function:

12. $f(x)=3(x-1)^2+5$

A.S: $x=1$

V: $(1,5)$

13. $f(x)=5(x-1)^2-7$

A.S: $x=1$

V: $(1,-7)$

Find the vertex and axis of symmetry for each function. Rewrite the equation in vertex form:

14. $f(x)=3x^2+5x-4$

$\frac{y+4}{3}+\frac{25}{36}=x^2+\frac{5}{3}x+\frac{25}{36}$

$\frac{y+4}{3}=\left(x+\frac{5}{6}\right)^2-\frac{25}{36}$

$y+4=3\left(x+\frac{5}{6}\right)^2-\frac{25}{12}$

$-4 \quad -\frac{48}{12}$

$y=3\left(x+\frac{5}{6}\right)^2-\frac{73}{12}$

15. $f(x)=8x-x^2+3$

$\frac{y-3}{-1}+16=x^2-8x+16$

$\frac{y-3}{-1}=(x-4)^2-16$

$y-3=-(x-4)^2+16$

$y=-(x-4)^2+19$

Use completing the square to describe the graph of each function.

16. $f(x)=x^2-4x+6$

$y-6=x^2-4x+4$

$+4$

$y-2=(x-2)^2$

$y=(x-2)^2+2$

17. $f(x)=10-16x-x^2$

$\frac{y-10}{-1}+64=x^2+16x+64$

$\frac{y-10}{-1}=(x+8)^2-64$

$y=-(x+8)^2+74$

18. $f(x)=2x^2+6x+7$

$\frac{y-7}{2}+\frac{9}{4}=x^2+3x+\frac{9}{4}$

$\frac{y-7}{2}=\left(x+\frac{3}{2}\right)^2-\frac{9}{4}$

$y-7=2\left(x+\frac{3}{2}\right)^2-\frac{9}{2}$

$+7 \quad +\frac{14}{2}$

$y=2\left(x+\frac{3}{2}\right)^2+\frac{5}{2}$

Write an equation for each parabola shown:

19.

20.