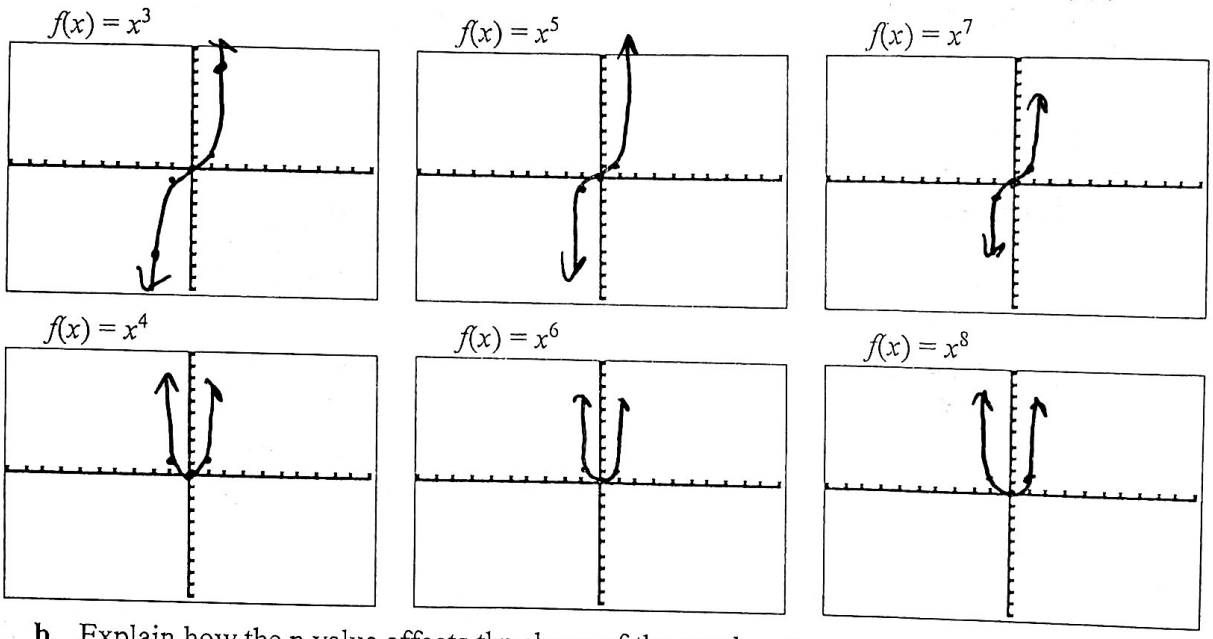
This exploration is about the graphs of two particular kinds of polynomials:

Power functions: A *power function* is a polynomial of the form $f(x) = x^n$.
We've previously studied x^2, but what about other powers: x^3, x^4, x^5, etc.
What can we find out about the graph of x^n in general?

Monomials: A *monomial* is a one-term polynomial, having the form $f(x) = a \cdot x^n$.
That is, a monomial is just a power function multiplied by a number a.
For example, $f(x) = 5x^3$ and $f(x) = -2x^4$ are power functions.
What do the graphs of these functions look like?

Investigation: graphs of power functions $f(x) = x^n$, n is an integer

1. a. Graph the functions shown below on your calculator, which are all of the form $f(x) = x^n$.



b. Explain how the n value affects the shape of the graph.

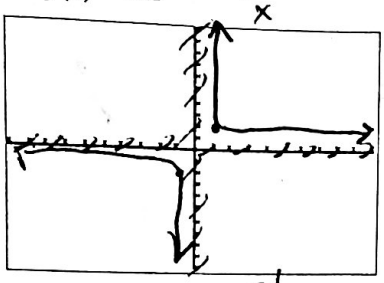
all evens are the same shape
all odds are the same shape

End Behavior
both: right side goes to infinity
even: left side matches right side
odd: left side is opposite right side

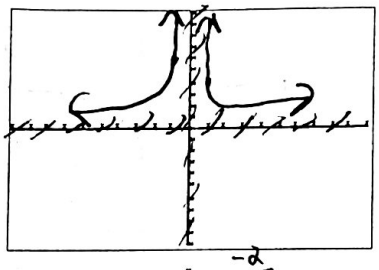
Investigations: graphs of monomials $f(x) = x^n$, n is a negative integer $\Rightarrow$ rational function

2. a. Graph the functions shown below on your calculator, which are all of the form $f(x) = ax^n$ where n is negative.

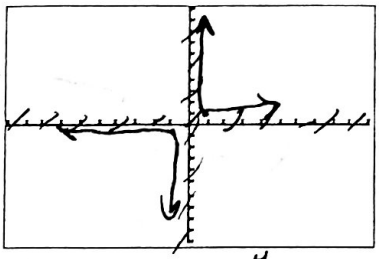
$f(x) = 2x^{-1} = \frac{2}{x}$



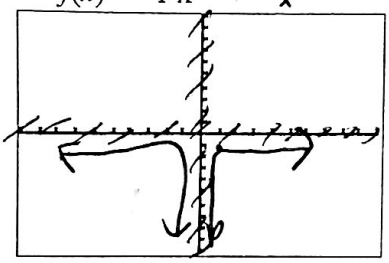
$f(x) = 4x^{-2}$ $\frac{4}{x^2}$



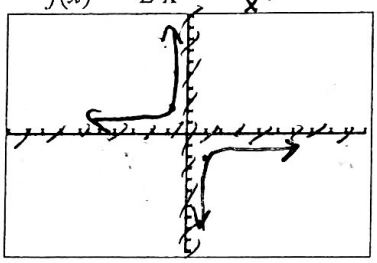
$f(x) = \frac{1}{2}\ x^{-3}$ $\frac{1}{2x^3}$



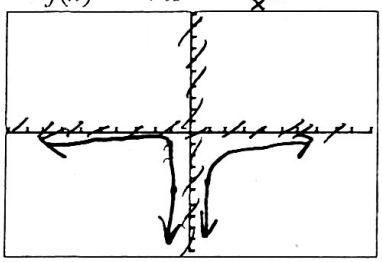
$f(x) = -1\ x^{-4} = \frac{-1}{x^4}$



$f(x) = -2\ x^{-5}$ $\frac{-2}{x^5}$



$f(x) = -4\ x^{-6}$ $\frac{-4}{x^6}$



b. Explain how the a value affects the shape of the graph.

vertical stretch or shrink
$-a$, reflects over x-axis

c. Explain how the n value affects the shape of the graph.

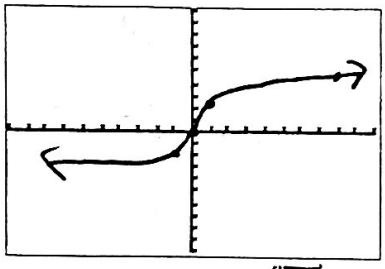
If n is even, symmetric to y-axis
(both top or both bottom)

If n is odd, symmetric to the origin
(opposite ~~corner~~ diagonal quadrants)
(Ex 1 & 3 or 2 & 4)

$f(x) = x^n$, n is a noninteger

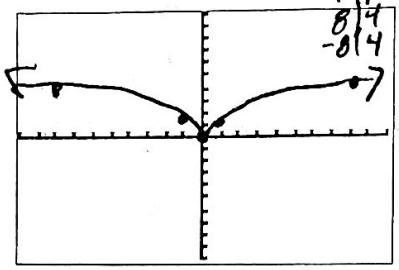
3. a. Graph the functions shown below on your calculator, which are all of the form $f(x) = ax^n$ where n is not an integer.

$f(x) = 2x^{1/3} = 2\sqrt[3]{x^1}$

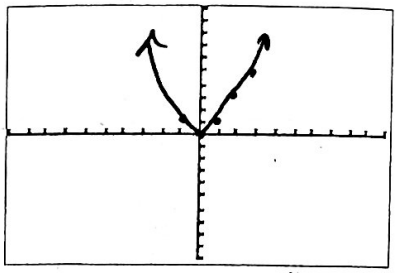


$f(x) = 4x^{2/3}$ $\sqrt[3]{x^2}$

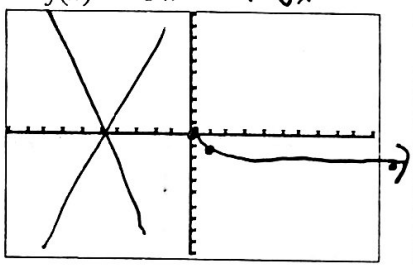
0	0
1	1
-1	1
8	4
-8	4



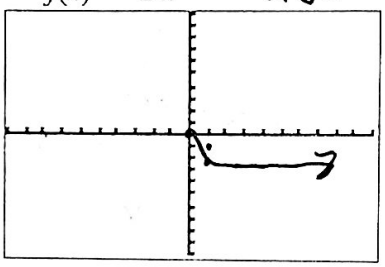
$f(x) = 1x^{4/3} = \sqrt[3]{x^4}$



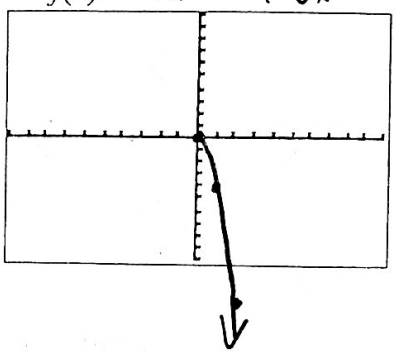
$f(x) = -1\,x^{1/4}$ $-1\sqrt[4]{x^1}$



$f(x) = -2\,x^{3/4} = -2\sqrt[4]{x^3}$



$f(x) = -4\,x^{7/4}$ $-4\sqrt[4]{x^7}$



b. Explain how the a value affects the shape of the graph.

vertical stretch or shrink

If a is −, reflects over x-axis

c. Explain how the n value affects the shape of the graph.

If n is <1, graph turns sideways

If n is >1, graph turns up or down (like a parabola)

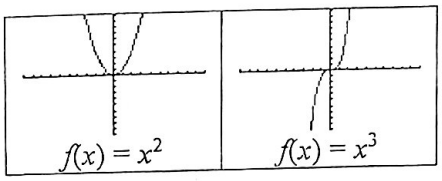
Summary: End Behavior

The term *end behavior* refers to whether each end of a graph goes up or down.
For example, the end behavior of $f(x) = x^3$ is: down on the left, up on the right.
Here is what can be seen about end behavior in the graphs you made in problems 1–3.

Right End Behavior: The right end behavior of $f(x) = ax^n$ depends on whether *a is* ± .

When *a is* + , the right end behavior is up

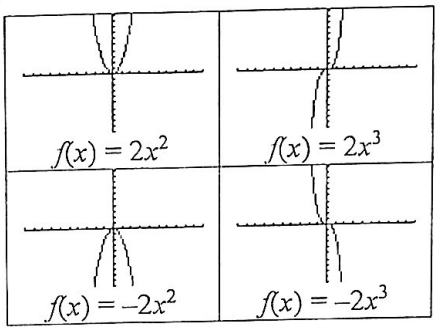
When *a is* − , the right end behavior is down



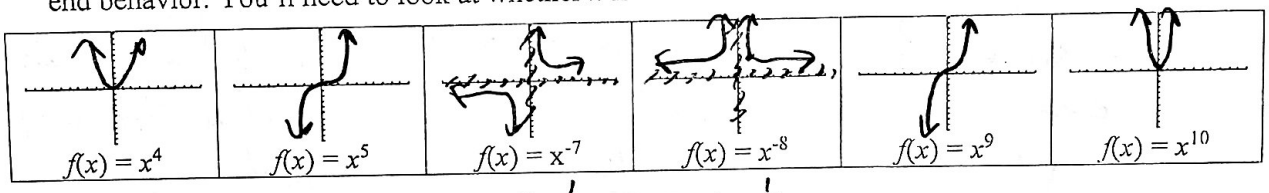
Left End Behavior: The left end behavior of $f(x) = ax^n$ depends on whether *n is* even or odd .

When *n is* even , the left end behavior is same as right end behavior

When *n is* odd , the left end behavior is opposite the right end behavior

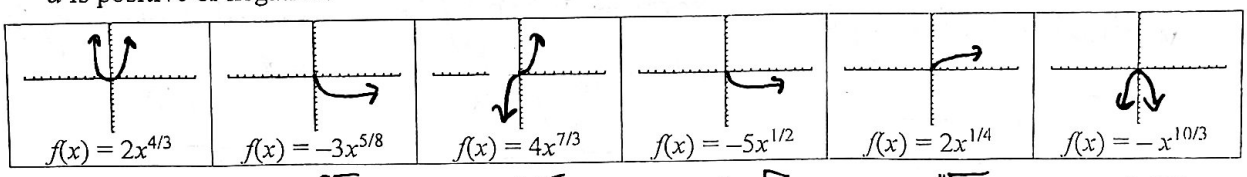


4. Without using a calculator and without looking back at earlier pages, make a rough sketch of the graph for each of the following functions of the form $f(x) = x^n$. Follow the rules about end behavior. You'll need to look at whether *n* is even or odd.



$= \frac{1}{x^7}$ $= \frac{1}{x^8}$

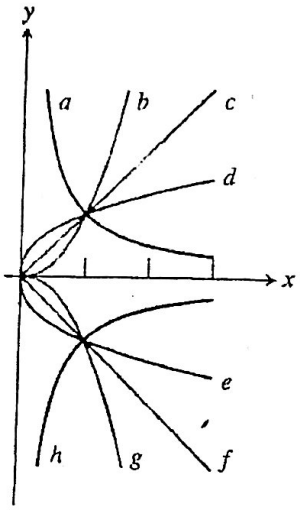
5. Again, without using a calculator, make a rough sketch of the graph for each of the following functions of the form $f(x) = x^n$. You'll need to look at whether *n* is even or odd and whether *a* is positive or negative.



$= 2\sqrt[3]{x^4}$ $-3\sqrt[8]{x^5}$ $4\sqrt[3]{x^7}$ $-5\sqrt{x}$ $2\sqrt[4]{x}$ $-\sqrt[3]{x^{10}}$

Precalculus Unit 2 Homework–Graphs of Power Functions

In exercises 1-6, match the equation to one of the curves labeled in the figure:



1. $f(x)=-\frac{2}{3}x^4$ G

2. $f(x)=\frac{1}{2}x^{-5}$ A

3. $f(x)=2x^{\frac{1}{4}}$ D

4. $f(x)=-x^{\frac{5}{3}}$ B G

5. $f(x)=-2x^{-2}$ H

6. $f(x)=1.7x^{\frac{2}{3}}$ D

In exercises 7-12, describe how to obtain the graph of the given monomial function from the graph of $g(x)=x^n$ with the same power n. State whether f is even or odd. Sketch the graph.

7. $f(x)=\frac{2}{3}x^4$

vert shrink by $\frac{2}{3}$

even

8. $f(x)=5x^3$

vertical stretch by 5

odd

9. $f(x)=-1.5x^5$

reflect over x-axis

vertical stretch by 1.5

odd

10. $f(x)=-2x^6$

reflect over x-axis

vertical stretch by 2

even

11. $f(x)=\frac{1}{4}x^8$

vertical shrink by $\frac{1}{4}$

even

12. $f(x)=\frac{1}{8}x^7$

vertical shrink by $\frac{1}{8}$

odd

13. True or False: The function $f(x)=x^{-\frac{2}{3}}$ is even. Justify your answer.

$\frac{1}{\sqrt[3]{x^2}}$ $f(1)=1$ $f(-1)=1$

14. True or False: The graph $f(x)=x^{\frac{1}{3}}$ is symmetric about the y-axis. Justify your answer.

$\sqrt[3]{x}$ → $y=\sqrt[3]{x}$

15. **Multiple-choice**: Let $f(x)=2x^{-\frac{1}{2}}$. What is the value of $f(4)$? $\frac{2}{\sqrt{x}}$

(a) 1 (b) -1 (c) $2\sqrt{2}$ (d) $\frac{1}{2\sqrt{2}}$ (e) 4

16. **Multiple-choice**: Let $f(x)=-3x^{-\frac{1}{3}}$. Which of the following statements is true? $\frac{-3}{\sqrt[3]{x}}$

(a) $f(0)=0$ (b) $f(-1)=-3$ (c) $f(1)=1$

(d) $f(3)=3$ (e) $f(0)$ is undefined

17. **Multiple-choice**: Let $f(x)=x^{\frac{2}{3}}$. Which of the following statements is true? $\sqrt[3]{x^2}$

(a) f is an odd function.

(b) f is an even function.

(c) f is neither even nor odd.

(d) The graph of f is symmetric with respect to the x-axis.

(e) The graph of f is symmetric with respect to the origin.

18. **Multiple-choice**: Which of the following is the domain of the function $f(x)=x^{\frac{3}{2}}$? $f(x)=\sqrt{x^3}$

(a) All reals

(b) $[0,\infty)$

(c) $(0,\infty)$

(d) $(-\infty,0)$

(e) $(-\infty,0)\cup(0,\infty)$