When dealing with simple **COMBINATIONS** of functions (such as $f+g, f-g, f \bullet g$, and $f \div g$), the domain of the resulting function consists of all numbers that belong to BOTH the domain of f and the domain of g.

$$(f+g)(x) = f(x) + g(x)$$

Example 1 Given that $f(x)=\sqrt{2-x}$ & $g(x)=\sqrt{2x+5}$, determine the value of $f+g$, $f-g$, $f \bullet g$, and $f \div g$ and their domains.

$f(x) = \sqrt{2-x}$

$D: x \leq 2$

$g(x) = \sqrt{2x+5}$

$D: x \geq \frac{-5}{2}$

$x \geq -2.5$

$(f+g)(x) = \sqrt{2-x} + \sqrt{2x+5}$

$D: [-2.5, 2]$

$(f-g)(x) = \sqrt{2-x} - \sqrt{2x+5}$

$D: [-2.5, 2]$

$(f \bullet g)(x) = \left(\sqrt{2-x}\right)\left(\sqrt{2x+5}\right)$

$D: [-2.5, 2]$

$(f \div g)(x) = \dfrac{\sqrt{2-x}}{\sqrt{2x+5}}$

$D: (-2.5, 2]$

Example 2 Perform the indicated operation, simplify your result as much as possible, and then determine the domain of the resulting function.

a) If $f(x)=\dfrac{4}{x-2}$ and $g(x)=\dfrac{-x^2}{x-2}$, find $(f+g)(x)$.

$$(f+g)(x) = \frac{4}{x-2} + \frac{-x^2}{x-2} = \frac{4-x^2}{x-2} = \frac{-1(x^2-4)}{x-2} = \frac{-1(x+2)\cancel{(x-2)}}{\cancel{x-2}} = \boxed{-1(x+2) \text{ or } -x-2}$$

$D: (-\infty, 2) \cup (2, \infty)$

b) If $f(x)=\sqrt{x-10}$ and $g(x)=\sqrt{x+10}$, find $(fg)(x)$.

$(f \cdot g)(x) = \sqrt{x-10} \cdot \sqrt{x+10}$

$x \geq 10$

$x \geq -10$

$D: [10, \infty)$

c) If $f(x)=\sqrt{x-5}$ and $g(x)=\sqrt{x+2}$, find $(f/g)(x)$.

$\left(\frac{f}{g}\right)(x) = \dfrac{\sqrt{x-5}}{\sqrt{x+2}}$

$x \geq 5$

$x \geq -2$

$x \neq -2$

$D: [5, \infty)$

When dealing with **COMPOSITIONS** of functions (such as $f \circ g$), the domain of the function consists of all x-values in the domain of g that map to $g(x)$ values in the domain of f.

of

$$(f \circ g)(x) = f(g(x))$$

Example 3 Given $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 1$. Find each composition function and its domain.

a) $f(g(x)) = \sqrt{(x^2+1)-1}$

$= \sqrt{x^2}$ $\quad D: (-\infty, \infty)$

$= x$

b) $g(f(x)) = (\sqrt{x-1})^2 + 1$ $\quad D: [1, \infty)$

$= x - 1 + 1$

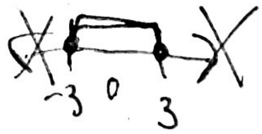
$= x$

Example 4 Given $f(x) = 9 - x^2$ and $g(x) = \sqrt{x}$. Find each composition function and its domain.

a) $f(g(x)) = 9 - (\sqrt{x})^2$ $\quad D: [0, \infty)$

$= 9 - x$

b) $g(f(x)) = \sqrt{9 - x^2}$



$\sqrt{-(x^2-9)}$ $\quad D: [-3, 3]$

$\sqrt{-(x-3)(x+3)}$

$\sqrt{(x+3)^2}$

Example 5 Given $f(x) = \dfrac{1}{x+3}$ and $g(x) = x^2 - 3$. Find each composition function and its domain.

a) $f(g(x)) = \dfrac{1}{(x^2-3)+3}$ $\quad (-\infty, 0) \cup (0, \infty)$

$= \dfrac{1}{x^2}$

b) $g(f(x)) = \left(\dfrac{1}{x+3}\right)^2 - 3$ $\quad (-\infty, -3) \cup (-3, \infty)$

$\dfrac{1}{(x+3)^2} - 3$

When **DECOMPOSING** functions the purpose is to create two functions (not using the identity function) such that their composition is the given function. In other words, when given $h(x)$ the goal is to define $f(x)$ & $g(x)$ so that $h(x) = f(g(x))$.

Example 6 For each $h(x)$, find the functions f and g such that $h(x) = f(g(x))$.

a) $h(x) = (x+1)^2 - 3(x+1) + 4$

$f(x) = x^2 - 3x + 4$ $g(x) = x + 1$

b) $h(x) = \sqrt[3]{2x+1}$

$f(x) = \sqrt[3]{x}$ $g(x) = 2x + 1$

or

$f(x) = \sqrt[3]{x+1}$ $g(x) = 2x$

c) $h(x) = \dfrac{x+5}{x^2+10x+25} = \dfrac{x+5}{(x+5)(x+5)}$

$h(x) = \dfrac{x+5}{(x+5)^2}$

$f(x) = \dfrac{x}{x^2}$ $g(x) = x + 5$

d) $h(x) = (9x^2 + 6x) - 2$

$f(x) = x - 2$ $g(x) = 9x^2 + 6x$

In #1-4, find the formulas for the functions $f+g$, $f-g$, and $f \cdot g$. Give the domain of each function along with the domain of f and g.

1. $f(x)=2x-1$ and $g(x)=x^2$
2. $f(x)=(x-1)^2$ and $g(x)=3-x$
3. $f(x)=\sqrt{x}$ and $g(x)=\sin x$
4. $f(x)=\sqrt{x+5}$ and $g(x)=|x+3|$

In #5-6, find formulas for $\dfrac{f}{g}$ and $\dfrac{g}{f}$. Give the domain of each function along with the domain of f and g.

5. $f(x)=\sqrt{x+3}$ and $g(x)=x^2$
6. $f(x)=\sqrt{x-2}$ and $g(x)=\sqrt{x+4}$

In #7-8, find $(f \circ g)(3)$ and $(g \circ f)(-2)$.

7. $f(x)=2x-3$ and $g(x)=x+1$
8. $f(x)=x^2-1$ and $g(x)=2x-3$

In #9-12, find $f(g(x))$ and $g(f(x))$. State the domain of each composition.

9. $f(x)=3x+2$ and $g(x)=x-1$
10. $f(x)=x^2-1$ and $g(x)=\dfrac{1}{x-1}$
11. $f(x)=x^2-2$ and $g(x)=\sqrt{x+1}$
12. $f(x)=\dfrac{1}{x-1}$ and $g(x)=\sqrt{x}$

In #13-18, find $f(x)$ and $g(x)$ so that the function can be described as $y = f(g(x))$.

13. $y = \sqrt{x^2 - 5x}$

14. $y = \left(x^3 + 1\right)^2$

15. $y = |3x - 2|$

16. $y = \dfrac{1}{x^3 - 5x + 3}$

17. $y = (x - 3)^5 + 2$

18. $y = e^{\sin x}$

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① $f(x)=2x-1$ $g(x)=x^2$

$D:(-\infty,\infty)$ $D:(-\infty,\infty)$

ⓐ $(f+g)(x)= 2x-1+x^2$

$D:(-\infty,\infty)$

ⓑ $(f-g)(x)=2x-1-x^2$

$D:(-\infty,\infty)$

ⓒ $(f\cdot g)(x)=(2x-1)(x^2)$

$=2x^3-x^2$

$D:(-\infty,\infty)$

ⓓ $f/g)(x)=\frac{2x-1}{x^2}$

$D:(-\infty,0)\cup(0,\infty)$

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③ $f(x)=\sqrt{x}$ $g(x)=\sin x$

$D:[0,\infty)$ $D:(-\infty,\infty)$

ⓐ $(f+g)(x)=\sqrt{x}+\sin x$

$D:[0,\infty)$

ⓑ $(f-g)(x)=\sqrt{x}-\sin x$

$D:[0,\infty)$

ⓒ $(f\cdot g)(x)=\sqrt{x}\cdot\sin x$

$D:[0,\infty)$

ⓓ $(f/g)(x)=\frac{\sqrt{x}}{\sin x}$

$D:(0,\pi)+\pi k$

② $f(x)=(x-1)^2$ $g(x)=3-x$

$D:(-\infty,\infty)$ $D:(-\infty,\infty)$

ⓐ $(f+g)(x)=(x-1)^2+3-x$

$x^2-2x+1+3-x$

x^2-3x+4

$D:(-\infty,\infty)$

ⓑ $(f-g)(x)=(x-1)^2-(3-x)$

$=x^2-2x+1-3+x$

$=x^2-x-2$

$D:(-\infty,\infty)$

ⓒ $(f\cdot g)(x)=(x-1)^2(3-x)$

$(x^2-2x+1)(3-x)$

$3x^2-6x+3-x^3+2x^2-x$

$-x^3+5x^2-7x+3$

$D:(-\infty,\infty)$

ⓓ $(f/g)(x)=\frac{(x-1)^2}{3-x}$

$D:(-\infty,3)\cup(3,\infty)$

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④ $f(x)=\sqrt{x+5}$ $g(x)=|x+3|$

$D:[-5,\infty)$ $D:(-\infty,\infty)$

ⓐ $(f+g)(x)=\sqrt{x+5}+|x+3|$

$D:[-5,\infty)$

ⓑ $(f-g)(x)=\sqrt{x+5}-|x+3|$

$D:[-5,\infty)$

ⓒ $(f\cdot g)(x)=(\sqrt{x+5})(|x+3|$

$D:[-5,\infty)$

ⓓ $(f/g)(x)=\frac{\sqrt{x+5}}{|x+3|}$

$D:[-5,-3)\cup(-3,\infty)$

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(5) $f(x) = \sqrt{x+3}$ $g(x) = x^2$

D: $[-3, \infty)$ D: $(-\infty, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+3}}{x^2}$

D: $[-3, 0) \cup (0, \infty)$

$\left(\frac{g}{f}\right)(x) = \frac{x^2}{\sqrt{x+3}}$

D: $(-3, \infty)$

(6) $f(x) = \sqrt{x-2}$ $g(x) = \sqrt{x+4}$

D: $[2, \infty)$ D: $[-4, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-2}}{\sqrt{x+4}}$

D: $[2, \infty)$

$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{x+4}}{\sqrt{x-2}}$

D: $(2, \infty)$

(7) $(f \circ g)(3)$ and $(g \circ f)(-2)$

$f(x) = 2x-3$ $g(x) = x+1$

$(f \circ g)(3) = 2(3+1)-3$

$= 5$

$(g \circ f)(-2) = (2 \cdot (-2) - 3) + 1$

$= -6$

(8) $f(x) = x^2 - 1$ $g(x) = 2x-3$

$(f \circ g)(3) = (2(3)-3)^2 - 1$

$= 8$

$(g \circ f)(-2) = 2((-2)^2 - 1) - 3$

$= 3$

(9) $f(x) = 3x+2$ $g(x) = x-1$

$f(g(x)) = 3(x-1)+2$

$= 3x - 3 + 2$

$= 3x - 1$

D: $(-\infty, \infty)$

$g(f(x)) = 3x+2-1$

$= 3x+1$

D: $(-\infty, \infty)$

(10) $f(x) = x^2 - 1$ $g(x) = \frac{1}{x-1}$

$f(g(x)) = \left(\frac{1}{x-1}\right)^2 - 1$

$= \frac{1}{(x-1)^2} - 1$

D: $(-\infty, 1) \cup (1, \infty)$

$g(f(x)) = \frac{1}{x^2-1-1}$

$= \frac{1}{x^2-2}$ $x^2 - 2 \neq 0$

$x^2 \neq 2$

D: $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$ $x \neq \pm\sqrt{2}$

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(11) $f(x)=x^2-2 \quad g(x)=\sqrt{x+1}$

$f(g(x))=(\sqrt{x+1})^2-2$ $\quad x+1\geq 0$

$= x+1-2$ $\quad x\geq -1$

$= x-1$

D: $[-1,\infty)$

$g(f(x))=\sqrt{x^2-2+1}$

$\sqrt{x^2-1}$ $\quad x^2-1\geq 0$

$x^2\geq 1$

D: $(-\infty,-1]\cup[1,\infty)$

(12) $f(x)=\frac{1}{x-1} \quad g(x)=\sqrt{x}$

$f(g(x))=\frac{1}{\sqrt{x}-1}$ $\quad x\geq 0$

$\sqrt{x}-1\neq 0$

$\sqrt{x}\neq 1$

$x\neq 1$

D: $[0,1)\cup(1,\infty)$

$g(f(x))=\sqrt{\frac{1}{x-1}}$

$=\frac{\sqrt{1}}{\sqrt{x-1}}$

$=\frac{1}{\sqrt{x-1}}$ $\quad x-1\geq 0 \quad \sqrt{x-1}\neq 0$

$x\geq 1 \quad x\neq 1$

D: $(1,\infty)$

(13) $y=\sqrt{x^2-5x}$

$f(x)=\sqrt{x} \quad g(x)=x^2-5x$

(14) $y=(x^3+1)^2$

$f(x)=x^2 \quad g(x)=x^3+1$

(15) $y=|3x-2|$

$f(x)=|x| \quad g(x)=3x-2$

(16) $y=\frac{1}{x^3-5x+3}$

$f(x)=\frac{1}{x} \quad g(x)=x^3-5x+3$

(17) $y=(x-3)^5+2$

$f(x)=x^5+2 \quad g(x)=x-3$

(18) $y=e^{\sin x}$

$f(x)=e^x \quad g(x)=\sin x$